COLBURN'S SERIES.—PART II.

COLBURN'S

Intellectual

ARITHMETIC.

Philadelphia:
H. COWPERTHWAIT & CO.
1860.

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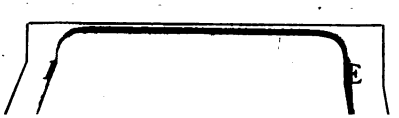

COLBURN'S SERIES. PART II.

Inductive Lessons

IN

INTELLECTUAL ARITHMETIC:

DESIGNED TO ILLUSTRATE

THE NATURE OF NUMBERS

AND OF

NUMERICAL OPERATIONS.

BY

DANA P. COLBURN,

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PHILADELPHIA:
H. COWPERTHWAIT & CO.
1860.

ELECTROTYPED BY MEARS & DUSENBERY.

PREFACE.

THIS book is designed for use in the various grades of schools above the Primary, in which Intellectual Arithmetic is made a subject of study.

The plan of instruction which it develops is believed to be strictly inductive, truly progressive, and thoroughly practical. Beginning with the smaller numbers and easier combinations, it first presents the operations of addition and subtraction, and then those of multiplication and division. It next treats of multiples and divisors, fractions, interest, profit and loss, partnership, and such other subjects pertaining to business life as the limits of the work will allow.

In every part of the book, the operations required are carefully illustrated and explained, and copious exercises are given to insure skill and readiness in their practical application.

Great care has been taken to make the solutions full and complete, and yet as concise as is consistent with mathematical precision and accuracy. They should receive careful attention from the pupil, for they require in their application that peculiar exercise of the reasoning powers which renders the study of the mathematics so valuable as a means of mental discipline.

The successive lessons are so constructed that each new step is either an easy advance or a logical deduction from those which precede, and are so arranged as to indicate the mutual connection and dependence of the various operations. The book, as a whole, thus becomes a connected treatise, the study of which will, it is thought, tend to develop the intellectual powers of the pupil, and to cultivate in him correct habits of mathematical investigation. In the hope that such may be the case, the Author would respectfully submit it to the consideration of those interested in education.

BRISTOL, R. I., Oct. 10, 1859.

Inductive Lessons

IN

INTELLECTUAL ARITHMETIC.

SECTION I.

A. A UNIT is a single thing or one.

ILLUSTRATION.—An apple is a unit; so is a book, a dollar, an ounce, a yard.

NUMBERS are used to show how many units there are in any given quantity.

Numbers are represented by FIGURES, as follows:—

1, or *1*, stands for one.	6, or *6*, stands for six.
2, or *2*, stands for two.	7, or *7*, stands for seven.
3, or *3*, stands for three.	8, or *8*, stands for eight.
4, or *4*, stands for four.	9, or *9*, stands for nine.
5, or *5*, stands for five.	10, or *10*, stands for ten.

B. 3 pens and 2 pens are how many pens?

2 books and 3 books are how many books?

4 and 3 are how many?

1. The last three questions are questions in ADDITION.

2. ADDITION is a process by which we ascertain how many units there are in two or more numbers taken together.

3. The result of the addition is called the SUM or AMOUNT.

ILLUSTRATION. The sum of 3 pens and 2 pens is 5 pens; the sum of 4 and 3 is 7.

4. The sign of addition is a cross made thus, +. It is called *plus* or *and*, and shows that the numbers between which it is placed are to be added together.

ILLUSTRATION. "4 + 3," means that 4 is to be added to 3. "2 + 5" are 7, means the same as 2 and 5 are 7

5. The sign of equality is formed by two parallel lines, thus =. It is called *equals*, or *is equal to*, and shows that the numbers or quantities between which it is placed are equal to each other.

ILLUSTRATION. "5 + 2 = 5," means that 3 and 2 are 5. It would be read, "3 plus 2 equals 5."

6. In this book, a star is often used in place of the words "How many."

ILLUSTRATION. "3 + * = 7?" means "2 plus how many are 7?"

C. How many are—

1. 4 + 5?
2. 5 + 4?
3. 2 + 4?
4. 3 + 7?
5. 8 + 2?
6. 2 + 8?
7. 5 + 2 + 3?
8. 3 + 3 + 3?
9. 2 + 4 + 2?

1. 4 + * = 9?
2. 5 + * = 9?
3. 6 + * = 10?
4. 3 + * = 7?
5. 3 + * = 8?
6. 4 + * = 10?
7. 5 + * = 7?
8. 4 + * = 6?

E. 1. How many are 4 from 6?

Ans. 4 from 6 leaves 2, because 4 and 2 are 6.

How many are—

2. 3 from 9?
3. 2 from 7?
4. 5 from 10?
5. 7 from 10?
6. 3 from 10?
7. 3 from 6?

F. 1. How many more are 7 than 3?

Ans. 7 = 4 more than 3, because 3 + 4 = 7.

How many more are—

2. 8 than 6?
3. 5 than 3?
4. 10 than 5?
5. 9 than 5?
6. 7 than 3?
7. 8 than 4?

G. 1. The questions under the last two letters are questions in SUBTRACTION.

2. SUBTRACTION is a process by which we find how many units there are in the difference of two numbers, or in the excess of one number over another.

3. The sign of subtraction is a line like a dash, thus, —. It is called *minus* or *less*, and signifies that the number after it is to be subtracted from the one before it.

ILLUSTRATION. "5 — 3 = 2," is read "5 minus 3 equals 2," or "5 less 3 equals 2." It means that 3 is to be subtracted from 5.

How many are—

1. 4 — 2?
2. 6 — 4?
3. 7 — 3?
4. 10 — 5?
5. 8 — 6?
6. 2 + 7 — 3?
7. 8 + 2 — 5?
8. 4 + 3 + 2 — 6?
9. 5 + 5 — 4 — 3?
10. 6 + 3 — 2 — 2?

H. 1. A man bought a hat for 4 dollars, and a cane for 2 dollars. How much did he pay for both?

SOLUTION. If he paid 4 dollars for the hat and 2 dollars for the cane, he must have paid 4 dollars plus 2 dollars, which are 6 dollars, for both.

2. I bought 4 bushels of white corn and 5 bushels of yellow corn. How many bushels of corn did I buy?

3. If I walk 5 miles in the forenoon, and 5 miles in the afternoon, how many miles shall I walk in both?

4. If a man pays 7 dollars for flour and 2 dollars for sugar, how much does he pay for both?

5. I bought a hat for 3 dollars, a pair of boots for 4 dollars, and a carpet-bag for 2 dollars. What was the amount of my purchase?

6. Mr. Brown has paid 2 dollars to one man, 3 dollars to another, 3 dollars to another, and 2 dollars to another. How many dollars has he paid to all?

7. A man bought a sheep for 5 dollars and sold it for 7 dollars. How many dollars did he gain?

SOLUTION. If he bought the sheep for 5 dollars and sold it for 7 dollars, he must have gained the difference between 5 dollars and 7 dollars, which is 2 dollars.

8. Arthur had 9 books, but he gave away 3 of them. How many did he have left?

9. If a farmer who has 10 cows should sell 5 of them, how many would he have left?

10. Edward has 9 marbles and Rufus has 6. How many more has Edward than Rufus?

11. A drover who had 8 oxen sold 2 of them to one man, 4 to another, and the rest to another. How many did he sell to the last man?

12. A man who had 4 dollars earned 4 dollars more, and *then spent 2 dollars.* How many dollars did he have left?

13. I paid 4 cents for candy, 2 cents for apples, and then had 2 cents left. How many cents did I have at first?

14. Mary had 3 apples. She found 6 more, and then gave Fanny 2, Susan 2, Jane 2, and Maria 2, after which she found 4. How many did she then have?

15. This morning I had 4 dollars. I have since sold some apples for 3 dollars, and some pears for 3 dollars. I have also paid 5 dollars for groceries, 2 dollars for meat, and 1 dollar for shoes, and have received 5 dollars for potatoes, and 2 dollars for peaches. How many dollars have I now?

16. After Mr. Smith had spent 6 dollars and earned 7 dollars, he had 10 dollars. How many had he at first?

17. A drover bought 5 cows and sold 7, when he had 2 left. How many did he have at first?

18. How many dollars has the man who will have 3 dollars after receiving 5 dollars, and paying out 4 dollars?

SECTION II.

A. 1 + 10 = eleven = 11 = *11.*

2 + 10 = twelve = 12 = *12.*

3 + 10 = thirteen = 13 = *13.*

4 + 10 = fourteen = 14 = *14.*

5 + 10 = fifteen = 15 = *15.*

6 + 10 = sixteen = 16 = *16.*

7 + 10 = seventeen = 17 = *17.*

8 + 10 = eighteen = 18 = *18.*

9 + 10 = nineteen = 19 = *19.*

10 + 10 = twenty = 20 = *20.*

B. How many are—

1. 6 + 6?
2. 8 + 7?
3. 7 + 8?
4. 5 + 6?
5. 8 + 3?
6. 3 + 8?
7. 3 + 4?
8. 13 + 4?
9. 3 + 14?
10. 1 + 6?
11. 11 + 6?
12. 1 + 16?
13. 3 + 7 + 4?
14. 2 + 6 + 6?
15. 3 + 5 + 9?
16. 4 + 5 + 3?
17. 2 + 6 + 8?
18. 4 + 5 + 5?
19. 2 + 9 + 4?
20. 7 + 9 + 3?
21. 8 + 4 + 3?
22. 4 + 6 + 10?
23. 6 + 7 + 7?
24. 8 + 6 + 3?

C.
1. 8 + * = 12?
2. 4 + * = 14?
3. 9 + * = 16?
4. 7 + * = 16?
5. 6 + 7 + * = 17?
6. 2 + 9 + * = 15?
7. 4 + 6 + * = 20?
8. 3 + 9 + * = 20?

D. How many are—

1. 15 — 8?
2. 12 — 5?
3. 17 — 3?
4. 16 — 8?
5. 15 — 9?
6. 18 — 6?
7. 10 — 6?
8. 18 — 16?
9. 10 — 7?
10. 20 — 7?
11. 20 — 17?
12. 13 — 9 — 2?
13. 17 — 9 — 1?
14. 20 — 10 — 3?
15. 14 — 7 — 2?
16. 16 — 9 — 4?
17. 15 — 3 — 6?
18. 20 — 8 — 7?
19. 19 — 13 — 4?
20. 20 — 11 — 7?
21. 20 — 9 — 5?
22. 17 — 13 — 4?

E.
1. 8 + 7 + 3 — 6 — 4 + 7 — 2?
2. 6 + 9 + 4 — 3 — 7 — 4 — 5 + 13?
3. 5 + 8 + 7 — 2 — 6 — 4 + 6 + * = 20?
4. 20 — 8 — 3 + 6 + 3 — 7 — 4 + * = 16?
5. 18 — 11 + 6 + 7 — 4 — 3 + 4 — * = 16?
6. 9 + 9 — 8 — 8 + 7 + 7 — 6 — 6 + * = 17?

F. 1. I gathered 6 bushels of apples from one tree, 5 from another, 4 from another, and 3 from another. How many did I gather from all?

2. I bought a coat for 11 dollars, a hat for 5 dollars, and a vest for 4 dollars. What was the total cost?

3. Mr. Ellis bought some cloth, giving in payment a ten-dollar bill, a five-dollar bill, a three-dollar bill, and a two-dollar bill. How many dollars did it cost?

4. A lady bought a silk dress for 13 dollars, and some calico for 3 dollars, giving in payment a twenty-dollar bill. How much money ought she to receive back?

5. William gave 8 cents for a writing-book, 6 cents for some ink, 3 cents for a pen-holder, and enough to make 19 cents for pens. How many cents did he spend for pens?

6. How much "change" ought a person to receive who should give 2 dimes in payment for a half-quire of paper worth 12 cents, and an inkstand worth 6 cents.

7. Charles owes me 3 dollars, Edwin owes me 4 dollars more than Charles, and Rufus owes me as much as Charles and Edwin together. How much does Edwin owe me? Does Rufus? How much do the three boys owe me?

8. A man walked 7 miles in the morning, 4 miles in the afternoon, and enough to make up 15 miles in the evening. How far did he walk in the evening?

9. Arthur has 3 marbles, James has 2 more than Arthur, Rufus has 4 more than James, and John has 7 less than Rufus. How many marbles has James? Has Rufus? Has John? How many have all the boys?

10. If you should perform 4 problems on Monday, 2 more on Tuesday than on Monday, 3 less on Wednesday than on Tuesday, 1 less on Thursday than on Wednesday, and 2 more on Friday than on Thursday, how many would you perform on each day, and how many in all?

11. Robert has 4 apples, George has 3 more than Robert, and Samuel has 6 less than Robert and George together. How many apples has each boy, and how many have all?

12. I collected 6 dollars of A, 3 more of B than of A, 7 dollars less of C than of B, and enough to make up 20 dollars of D. How many dollars did I collect of D?

13. Francis bought some candy for 11 cents. He sold some of it to Harry for 4 cents, some to Luther for 5 cents, and the rest to Ambrose for such a sum that he gained 6 cents on the whole. How much did he receive for what he sold to Ambrose?

14. A man started from Providence and travelled 4 miles north, then 5 miles north, then 6 miles south, then 8 miles south, and then 3 miles south. How many miles was he from Providence, and in which direction?

15. A man started from Philadelphia and travelled 9 miles east, then 12 miles west, then 7 miles east, and then 5 miles west. How far was he from Philadelphia, and in which direction?

16. On Monday morning the mercury in the thermometer was 15 degrees above zero; on Tuesday it was 20 degrees lower than on Monday, and on Wednesday it was 12 degrees higher than on Tuesday. How high was it on Wednesday?

17. After paying 6 dollars for groceries and 3 dollars for cloth, I had 2 dollars less than I had paid away. How many dollars did I have at first?

18. I sold 3 lamps to Mr. Mason, 2 more to Mr. Arnold than to Mr. Mason, and still had 2 more than I sold. How many did I sell, and how many did I have left?

19. How many dollars must I have in order that after paying out 7 dollars, receiving 9 dollars, and losing 6 dollars, I may have 12 dollars?

20. If I should start from home and travel 5 miles

north, then 8 miles south, then 10 miles north, and then 5 miles south, how far should I be from home, and which way?

SECTION III.

A. 2 tens = twenty = 20, or 20.

3 tens = thirty = 30, or 30.

4 tens = forty = 40, or 40.

5 tens = fifty = 50, or 50.

6 tens = sixty = 60, or 60.

7 tens = seventy = 70, or 70.

8 tens = eighty = 80, or 80.

9 tens = ninety = 90, or 90.

10 tens = one hundred = 100, or 100.

B. How many tens are equal

1. To 50?	4. To 70?	7. To 90?
2. To 80?	5. To 20?	8. To 40?
3. To 30?	6. To 60?	9. To 100?

C. What number is equal

1. To 3 tens?	4. To 4 tens?	7. To 6 tens?
2. To 9 tens?	5. To 8 tens?	8. To 2 tens?
3. To 10 tens?	6. To 7 tens?	9. To 5 tens?

D. Write each of the following numbers in figures.

1. Forty.	4. Ninety.	7. Thirty
2. Eighty.	5. Seventy.	8. Sixty.
3. Twenty.	6. Fifty.	9. One hundred.

E. 1. 4 tens + 5 tens = * tens? Then 40 + 50 = *?
2. 4 tens + 4 tens = * tens? Then 40 + 40 = *?
3. 7 tens + 3 tens = * tens? Then 70 + 30 = *?
4. 6 tens + * tens = 9 tens? Then 60 + * = 90?
5. 4 tens + * tens = 10 tens? Then 40 + * = 100?
6. 2 tens + * tens = 6 tens? Then 20 + * = 60?
7. 8 tens — 5 tens = * tens? Then 80 — 50 = *?
8. 4 tens — 3 tens = * tens? Then 40 — 30 = *?
9. 10 tens — 3 tens = * tens? Then 100 — 30 = *?

F. How many are—

1. 50 + 40?
2. 20 + 30?
3. 50 + 50?
4. 20 + 70?
5. 50 — 20?
6. 70 — 30?
7. 100 — 80?
8. 90 — 60?
9. 40 + 60?
10. 30 + 70?
11. 100 — 10?
12. 80 + 40?
13. 40 + 30 + 20?
14. 50 + 20 + 30?
15. 20 + 30 + 30?
16. 20 + 40 + 20?
17. 40 + 20 + 30?
18. 30 + 30 + 40?
19. 90 — 20 — 40?
20. 100 — 60 — 30?
21. 90 — 20 — 20 — 20 — 20?
22. 30 + 30 + 40 — 50 — 20?
23. 20 + 20 + 20 + 20 + 20 — 70?
24. 100 — 20 — 20 — 20 — 20 + 50?

G. 1. A man gave 20 cents for "Harper's Monthly," 20 for the "Atlantic," and 50 for the "North American Review." How many cents did he pay for all?

2. George gathered 20 quarts of nuts Jabez gathered 30 quarts, and Freeman gathered as many as George and Jabez together. How many quarts did all of them gather?

3. A farmer who owned 90 sheep, kept 40 in one pasture, 30 in another, and the rest in another. How many did he keep in the last pasture?

4. Mr. Smith had 30 apple-trees, 40 peach-trees, and 20 pear-trees, but 20 of his peach-trees and 10 of his pear-trees were destroyed. How many trees had he left?

5. *Mr. Wilson* went a journey of 100 miles in 3 days.

The first day he rode 30 miles, and the second day 10 more than he did the first. How many miles did he ride the third day?

6. Mr. Tyler owns a wheat-field of 10 acres, a rye-field of 10 acres, a corn-field of 20 acres, and a pasture of 50 acres. How many acres of land does he own in all? How many acres would he own if he should exchange his wheat-field for 20 acres of wood-land?

7. Richard has 20 cents, Horace has 20 cents, and Nathan has 20 cents more than Richard and Horace together. How many cents have all the boys?

8. If the boys named in the last problem should put their money together, and buy some paper for 20 cents, some pencils for 20 cents, some pens for 10 cents, and a book for the remainder, how much would the book cost?

9. Mr. Rogers bought 40 barrels of apples from one man, 30 from another, and 20 from another. He then sold 50 barrels to one man, and 10 to another. How many did he have left?

10. I had 100 dollars. I paid 20 dollars to Mr. Austin, 40 dollars to Mr. Perry, and so much to Mr. Richards, that I had but 10 dollars left. How many dollars did I pay to Mr. Richards?

11. Mr. Cole owes me 10 dollars, Mr. Gray owes me 10 dollars more than Mr. Cole, and Mr. Rich owes me 10 dollars more than Mr. Cole and Mr. Gray together. How much do all owe me?

12. Mr. Arnold owes me 20 dollars more than Mr. Luther, and Mr. Luther owes me 10 dollars less than Mr. Brown, and 20 dollars more than Mr. Gay. If Mr. Gay owes me 20 dollars, how much does each of the others owe me? How much do the three together owe me?

13. George can spend 30 dollars and still have 30 dollars

left. William has 10 dollars less than George, and 30 dollars more than Asa. How many dollars has each of the boys?

14. Reuben lacks but 20 dollars of having money enough to buy a watch for 70 dollars, but if Walter should give him his money, he could buy the watch and have 20 dollars left. How many dollars has each of the boys?

15. George and Frank together have money enough to buy a book for 30 cents, a knife for 20 cents, and a box for 30 cents; George alone has 10 cents more than money enough to buy the book. How many cents has Frank?

SECTION IV.

A. 2 tens + 1 unit = twenty-one = 21, or *21*.

2 tens + 2 units = twenty-two = 22, or *22*.

2 tens + 9 units = twenty-nine = 29, or *23*.

3 tens + 1 unit = thirty-one = 31, or *24*.

8 tens + 7 units = eighty-seven = 87, or *25*.

9 tens + 9 units = ninety-nine = 99, or *26*.

B. Count from twenty to one hundred, thus: Twenty-one, twenty-two, twenty-three, &c. &c.

What is the value of

1. 20 + 8?	3. 30 + 7?	5. 60 + 9?
2. 40 + 6?	4. 90 + 3?	6. 20 + 5?

C. How many tens and units are equal

1. To 64?	4. To 28?	7. To 23?
2. To 37?	5. To 50?	8. To 19?
3. To 73?	6. To 17?	9. To 91?

D. Write each of the following numbers :—

1. Seventy-nine.	4. Fifty-seven.	7. Eighty-six.
2. Twenty-four.	5. Sixty-nine.	8. One hundred.
3. Forty-two.	6. Ninety-six.	9. Fifty-four.

E. How many are—

1. 4 + 2?	11. 4 + 2 + 2?
2. 24 + 2?	12. 24 + 2 + 2?
3. 64 + 2?	13. 84 + 2 + 2?
4. 7 + 3?	14. 3 + 2 + 1 + 4?
5. 27 + 3?	15. 23 + 2 + 1 + 4?
6. 57 + 3?	16. 73 + 2 + 1 + 4?
7. 84 + 3?	17. 63 + 7 + 5 + 2 + 3?
8. 22 + 6?	18. 81 + 5 + 4 + 6 + 3?
9. 57 + 3?	19. 85 + 5 + 7 + 3 + 6?
10. 95 + 5?	20. 42 + 4 + 4 + 3 + 5?

F. How many are—

1. 2 + 5 + 3 + 3 + 2 + 5 + 3 + 5 + 2 + 3?
2. 2 + 7 + 1 + 2 + 7 + 1 + 2 + 7 + 1 + 2?
3. 3 + 3 + 4 + 3 + 3 + 4 + 3 + 3 + 4 + 3?
4. 53 + 3 + 4 + 3 + 3 + 4 + 3 + 3 + 4 + 3?
5. 24 + 2 + 4 + 1 + 7 + 2 + 4 + 6 + 2 + 5?
6. 35 + 3 + 2 + 3 + 7 + 1 + 9 + 2 + 4 + 2?
7. 42 + 8 + 5 + 3 + 2 + 4 + 6 + 2 + 7 + 1?
8. 57 + 3 + 1 + 3 + 5 + 1 + 2 + 3 + 4 + 1?
9. 14 + 6 + 4 + 6 + 3 + 7 + 5 + 5 + 4 + 4?

G. How many are—

1. 8 — 3?	11. 9 — 2 — 5?
2. 28 — 3?	12. 49 — 2 — 5?
3. 98 — 3?	13. 99 — 2 — 5?
4. 10 — 6?	14. 90 — 3 — 7 — 6?
5. 40 — 6?	15. 50 — 3 — 7 — 6?
6. 100 — 6?	16. 100 — 3 — 7 — 6 — 2?
7. 47 — 6?	17. 94 — 4 — 3 — 2 — 4?
8. 69 — 4?	18. 67 — 5 — 2 — 4 — 6?
9. 80 — 7?	19. 100 — 6 — 4 — 5 — 3?
10. 50 — 9?	20. 46 — 4 — 2 — 3 — 6?

H. How many are—

1. 80 — 2 — 4 — 4 — 2 — 4 — 4 — 2?
2. 40 — 8 — 2 — 5 — 5 — 2 — 4 — 3?
3. 70 — 6 — 4 — 2 — 8 — 3 — 5 — 2?
4. 100 — 3 — 4 — 3 — 10 — 5 — 5 — 3?
5. 25 + 3 + 2 + 8 — 5 — 3 — 6 — 4?
6. 83 + 7 + 2 + 8 — 3 — 7 — 5 — 5?

I. **1.** A drover had 47 cattle. He sold 3 and bought 6, after which he sold 8 and bought 5. How many had he then?

2. Stephen had 43 walnuts, and his brother gave him 7 more, and his father gave him enough to make up 100. How many did his father give him?

3. A merchant bought some goods for 21 dollars, and sold them so as to gain 12 dollars less than they cost him. How many dollars did he receive for them?

4. What number is that to which if 3 be added the sum will be 80?

5. From what number must 7 be subtracted to leave a remainder of 52?

6. If 7 be added to a certain number, and 4 be subtracted from the sum, the result will be 5 less than the difference between 30 and 50. What is the number?

7. After paying out 10 dollars and receiving 5, I have 35 dollars. How many dollars did I have at first?

8. After walking 8 miles towards Boston, I walked 6 miles in the opposite direction, when I was just 38 miles from Boston. How far from it was I at first?

9. Sarah had 50 cents. She spent 10 cents for ribbon, 4 cents for sewing-silk, and 5 cents for needles. How many cents had she left?

10. Eliza found 15 shells in one place, 5 in another, 3 in *another, 7 in another,* and when she had found some in

another, she could give away 3, and still have 37 left. How many did she find in the last place?

11. I bought a wagon for 71 dollars, and paid 9 dollars for repairing it, 5 dollars for painting it, and so much for varnishing it that I lost 7 dollars by selling it for 82 dollars. How much did I pay for varnishing it?

12. A man bought a handkerchief for 25 cents, some sewing-silk for 5 cents, some thread for 7 cents, and some buttons for 3 cents, giving in payment a dollar bill. How much change ought he to receive back?

13. Erastus and Edwin played "odd or even," beginning their game with 20 grains of corn apiece. Erastus won 3 grains from Edwin, then he won 3 more, and then he won 4 more. How many had each boy then?

14. The boys named in the last problem played till Edwin had 24 grains of corn. How many did Erastus then have?

15. Mr. Day had 30 dollars. He paid 7 dollars to a shoemaker, 3 dollars to a laborer, and 5 dollars to a hatter. He then received 5 dollars from Mr. Baker, 30 dollars from Mr. Smith, and 8 dollars from Mr. Talbot, after which he gave Mr. Gay 6 dollars for groceries, and 2 dollars for cloth, and Dr. Fogg 7 dollars for services as a physician. How much money had he left?

LESSON V.

A. How many are—

1.	$9 + 8$?	10.	$5 + 6$?
2.	$19 + 8$?	11.	$45 + 6$?
3.	$49 + 8$?	12.	$65 + 6$?
4.	$6 + 9$?	13.	$8 + 4$?
5.	$36 + 9$?	14.	$48 + 4$?
6.	$6 + 89$?	15.	$8 + 74$?

How many are—

B. 1. $6 + 8 + 9$?
2. $16 + 8 + 9$?
3. $66 + 8 + 9$?
4. $86 + 8 + 9$?
5. $9 + 7 + 8$?
6. $39 + 7 + 8$?
7. $9 + 77 + 8$?
8. $9 + 47 + 8$?
9. $6 + * = 14$?
10. $6 + * = 84$?
11. $46 + * = 54$?
12. $76 + * = 84$?
13. $3 + * = 11$?
14. $13 + * = 21$?
15. $3 + * = 61$?
16. $3 + * = 91$?

How many are—

C. 1. $13 - 8$?
2. $23 - 8$?
3. $93 - 8$?
4. $14 - 6$?
5. $44 - 6$?
6. $94 - 6$?
7. $11 - 5$?
8. $81 - 5$?
9. $41 - 5$?
10. $12 - 9$?
11. $42 - 9$?
12. $92 - 9$?

To the Teacher.—Vary and extend the preceding exercises till the scholars appreciate the connection between $9 + 8$, $19 + 8$, $29 + 8$, &c., and understand fully that as $9 + 8 = 17$, so $49 + 8 = 57$, $79 + 8 = 87$, &c.; that as $13 - 9 = 4$, so $23 - 9 = 14$, $83 - 9 = 74$, &c. The great objects to be aimed at are accuracy and promptness, the latter being scarcely less important than the former.

How many are—

D. 1. $48 + 4$?
2. $37 + 6$?
3. $29 + 7$?
4. $53 + 8$?
5. $74 + 7$?
6. $5 + 69$?
7. $46 + 8$?
8. $6 + 48$?
9. $9 + 79$?
10. $4 + 67$?
11. $23 + 9 + 5 + 8$?
12. $27 + 6 + 8 + 5$?
13. $34 + 8 + 9 + 7 + 6$?
14. $26 + 3 + 5 + 6 + 5$?
15. $25 + 8 + 4 + 9 + 9$?
16. $29 + 7 + 9 + 4 - 4$?
17. $73 + 9 + 6 + 9 - 8$?
18. $28 + 5 + 8 + 7 - 9$?
19. $49 + 7 + 6 - 8 - 5$?
20. $67 + 8 + 8 - 9 - 7$?

21. $33 + 8 + 4 + 9 + 7 + 6 + 5 + 3$?
22. $56 + 6 + 9 + 3 + 8 + 5 + 4 + 6$?
23. $45 + 8 + 9 + 7 + 4 + 9 + 6 + 5$?
24. $38 + 6 + 7 + 7 + 5 + 8 + 9 + 8$?
25. $53 + 8 + 5 + 7 + 8 + 5 + 9 + 6$?
26. $48 + 5 + 9 + 6 + 5 - 6 + 9 - 4$?
27. $29 + 8 + 7 + 6 + 5 - 6 - 7 - 8$?
28. $67 + 4 + 5 + 6 + 7 + 8 + 9 + 8$?
29. $36 + 7 + 7 + 50 + 8 + 7 + 6 + 9$?
30. $100 + 9 + 7 + 6 + 8 + 5 + 9 + 7$?

E. **1.** Count by two's from 2 to 40, and back to 2 again.

Ans. Two, four, six, eight, ten, &c. to 40; then 40, 38, 36, 34, &c.

2. Count by 2's from 1 to 39, and back to 1 again.

Ans. One, three, five, seven, &c., to 39; then 39, 37, 35, &c.

3. Count by 3's from 3 to 36, and back; then from 1 to 40 and back; and then from 2 to 35 and back.

4. Count by 4's from 4 to 40 and back; from 1 to 37 and back; from 3 to 43 and back; from 2 to 38 and back.

5. Count by 5's from 5 to 50; from 2 to 37; from 4 to 49; from 1 to 56; and from 3 to 43; counting back in each case.

6. Count by 6's from 6 to 72; from 3 to 63; from 1 to 67; from 5 to 59; from 2 to 56; from 4 to 64; counting back in each case.

7. Count by 7's from 7 to 84; from 1 to 64; from 4 to 74; from 2 to 65; from 5 to 75; from 6 to 90, and from 3 to 80; counting back in each case.

8. Count by 8's from 8 to 96; from 7 to 87; from 2 to 90; from 4 to 76, from 6 to 78; from 5 to 69; from 3 to 91; counting back in each case.

9. Count by 9's from 9 to 99; from 6 to 87; from 3 to 93; *from 2 to* 74; from 5 to 86; from 8 to 98; from 7 to

79; from 4 to 85; from 1 to 100; counting back in each case.

10. Count by 10's from 10 to 100; from 7 to 97; from 4 to 94; from 1 to 91; from 2 to 92; from 5 to 95; from 8 to 98; from 9 to 99; from 3 to 93; from 6 to 96; counting back in each case.

NOTE.—The above exercises are very valuable, and if mastered, will give an almost perfect command over the processes of addition and subtraction. The class should practise on one or two of them daily, in connection with the recitation from other parts of the book. Such a course would usually be more profitable than to practise upon them exclusively till all are mastered. To add to their interest, sometimes let the pupils count in concert; sometimes let one scholar name one number, another the next, and so on; sometimes let different scholars try to see which can count the fastest without making a mistake; sometimes let one pupil begin with one number, and another at the same time with another number, and see if they can count in perfect time, and without "putting one another out;" sometimes let the class write the numbers on their slates, and read them in the class. The ingenious teacher will find no difficulty in interesting his pupils in these exercises, *a few minutes at a time*, till they are mastered.

1. In a school of 63 scholars, all but 8 write. How many write?

2. Jabez bought 37 filberts, found 6, gave away 9, and after eating some had 29 left. How many did he eat?

3. Henry had 42 cents. He earned 9 cents, 5 cents, and 8 cents, after which he spent 7 cents. How many cents had he left?

4. A farmer sold 57 bushels of corn to one man, 9 bushels to another, and enough to make up 71 bushels to another. How many did he sell to the last man?

5. Martin had 54 marbles. He lost 9, gave away 7, and

when his father had given him some more he had 43. How many did his father give him?

6. I sold a cow for 25 dollars, thereby losing 8 dollars. What did she cost me?

7. I bought a horse, and after letting him enough to come to 7 dollars, I sold him for 87 dollars. Allowing that I lost 6 dollars by the transaction, what did he cost me?

8. I bought a horse for 75 dollars. I have let him enough to amount to 10 dollars, and have paid 7 dollars for keeping him. For what must I sell him to make a clear gain of 10 dollars?

9. A man bought a horse for 63 dollars, and was obliged to sell him for 8 dollars less than cost. For how much did he sell him?

10. A farmer who had 33 bushels of corn, sold 6 bushels for 5 dollars. How many bushels had he left?

11. Mr. Adams owned 46 acres of land, and bought enough to make up 54 acres. How many did he buy?

12. A trader bought 8 yards of cloth, for which he paid 13 dollars, 6 yards for which he paid 9 dollars, 9 yards for which he paid 8 dollars, and 6 yards for which he paid 8 dollars. How many yards of cloth did he buy in all? How many dollars did he pay for it?

13. Sarah had 32 roses. She gave 9 to one companion, 8 to another, and, when she had given some to another, she had 7 left. How many did she give to the last?

14. I bought a lot of grain for 54 dollars, and paid 3 dollars to have it carried to my store. For how much must I sell it to gain 48 dollars less than its entire cost?

15. Mr. Jones paid 9 dollars to one man, 7 to another, and 8 to another, and had money enough left to buy a coat for 9 dollars, a hat for 8 dollars, and still have a five-dollar bill in his pocket. How much money did he have at first?

16. A man who lived just 50 miles west of Philade started from his house and travelled 6 miles east, tl miles west, then 4 miles west, and lastly 8 miles west. far was he then from Philadelphia?

17. Daniel has 58 cents, Warren has 9 more than D and if the difference between Daniel's money and Wa be added to Warren's, the sum will equal Austin's m How many cents has Austin?

18. Arthur says that he has 6 cents less than Ri and 9 cents more than Albert, and that he has 10 more than the difference between Albert's money Richard's. How many cents has each of the boys?

19. A farmer bought 51 sheep, and when he had s he had 49 left How many did he have at first?

20. A man bought a horse for 72 dollars, and after ing 4 dollars for keeping him, and letting him enou come to 7 dollars, he sold him for 70 dollars. Did he or lose, and how much?

LESSON VI.

A. How many are—

1. 30 + 50?	7. 40 + 20 + 20?
2. 32 + 50?	8. 43 + 20 + 20?
3. 39 + 50?	9. 40 + 28 + 20?
4. 20 + 60?	10. 30 + 20 + 40?
5. 25 + 60?	11. 38 + 20 + 40?
6. 20 + 68?	12. 30 + 20 + 47?

B. 1. How many are 24 + 67?

Solution. 24 + 60 are 84, + 7 are 91.

How many are—

2. 63 + 29?
3. 26 + 55?
4. 37 + 48?
5. 24 + 37?
6. 73 + 19?
7. 28 + 53?
8. 37 + 67?
9. 29 + 29?
10. 27 + 58 + 12?
11. 33 + 47 + 16?
12. 24 + 29 + 47?
13. 24 + 29 + 37?
14. 27 + 27 + 27?
15. 11 + 46 + 25?
16. 34 + 26 + 27?
17. 16 + 17 + 19?

C. How many are—

1. 80 — 20?
2. 86 — 20?
3. 60 — 30?
4. 67 — 30?
5. 70 — 40?
6. 77 — 40?
7. 70 — 30 — 20?
8. 75 — 30 — 20?
9. 90 — 40 — 30?
10. 91 — 40 — 30?
11. 80 — 20 — 40?
12. 83 — 20 — 40?

D. 1. How many are 68 — 26?

SOLUTION. 68 — 20 are 48, — 6 are 42.

How many are—

2. 43 — 17?
3. 92 — 67?
4. 83 — 48?
5. 61 — 23?
6. 81 — 23?
7. 52 — 27?
8. 48 — 29?
9. 97 — 58?
10. 47 — 19?
11. 82 — 46?
12. 64 — 37?
13. 96 — 78?

E. 1. Joseph bought an "Intellectual Arithmetic" for 25 cents, and a slate for 13 cents. How much did he pay for both?

2. Martha had 75 cents. She bought a Primary Geography for 37 cents, a Spelling Book for 17 cents, and spent the rest of her money for paper. How much did she spend for paper?

3. A farmer sold a horse for 93 dollars, which was 26 dollars *more than* he cost. How much did he cost?

4. A horse dealer bought a horse for 54 dollars, and after paying 17 dollars for keeping him, he sold him for 96 dollars. How much did he gain by the transaction?

5. I have 27 dollars. How much more money must I get to buy a watch for 73 dollars, and a chain for 12 dollars?

6. A man starting from home travelled 35 miles east, then 54 miles west, then 28 miles west, and then 73 miles east. How far was he from home, and which way?

7. Mr. Simmons started from Providence for Boston, and Mr. Bacon at the same time started from Boston for Providence, the distance between the cities being 42 miles. How far apart will they be when Mr. Simmons has travelled 19 miles and Mr. Bacon 14 miles?

8. How far apart will the men named in the last example be, when Mr. Simmons has travelled 29 miles, and Mr. Bacon 28 miles?

9. A man bought a sleigh for 24 dollars. He paid 9 dollars for painting and repairing it, and then gave it and 18 dollars in money for another sleigh. How much did the second sleigh cost him?

10. From a cask containing 64 gallons of oil, 18 gallons were drawn out at one time, and 25 at another, after which 17 gallons were put in. How many gallons were then in the cask?

11. There were 18 sheep in one flock, 27 in another, and 39 in another. How many were there in all?

12. Twenty-three of the sheep named in the last problem were sold to one man, 26 to another, and the rest to another. How many were sold to the last man?

13. Ralph shot 27 pigeons, 15 partridges, 14 woodcocks, and as many quails as there were partridges and woodcocks together. How many quails did he shoot? How many *birds* in all?

14. Mr. Thompson owes 13 dollars to Mr. Baker, 9 dollars more to Mr. Ellis than to Mr. Baker, and as much to Mr. French as he owes to Mr. Thompson and Mr. Ellis together. How much does he owe to each, and how much to all?

15. Mr. Talbot says that he has 17 dollars in silver, 25 in gold, and 13 dollars more in bank-bills than in silver and gold together, and that he will give you the difference between his money and 100 dollars, if you will tell him how many dollars he has in all. What will you tell him, and how much must he give you?

16. Carlos has 47 books, which is 29 more than Herman has. How many has Herman? How many have both?

17. Bernard has 33 books, which is 17 more than Amos has, and 19 less than Harry has. How many books has each of the boys, and how many have all?

SECTION VII.

A. The numbers above one hundred are counted thus:—

One hundred one, one hundred two, &c., one hundred ninety-nine, two hundred, two hundred one, two hundred two, &c., to ten hundred, which is generally called one thousand, one thousand and one, one thousand and two, &c., to eleven hundred and one, or one thousand one hundred and one, &c., to nineteen hundred ninety-nine or one thousand nine hundred ninety-nine, twenty hundred or two thousand, &c.

Ten hundred, or one thousand, is written 1000. Twenty hundred, or two thousand, is written 2000. Ninety hundred, or nine thousand, is written 9000.

The following exercises suggest the manner of reading and writing numbers above one hundred:—

100 + 2 = 102.	1000 + 10 = 1010.
100 + 9 = 109.	1100 + 3 = 1103.
100 + 10 = 110.	1000 + 28 = 1028.
100 + 11 = 111.	1100 + 11 = 1111.
300 + 12 = 312.	1100 + 17 = 1117.
600 + 20 = 620.	1000 + 117 = 1117.
800 + 29 = 829.	3200 + 20 = 3220.
1000 + 1 = 1001.	4200 + 34 = 4234.
1000 + 4 = 1004.	4000 + 234 = 4234.

B. Read the following numbers:—

1. 427.	5. 514.	9. 1728.
2. 168.	6. 2107.	10. 6006.
3. 250.	7. 1526.	11. 2224.
4. 630.	8. 3794.	12. 1347.

C. Write each of the following in figures:—

1. Three hundred twenty-seven.
2. Eight hundred four.
3. Seventeen hundred twenty-eight.
4. Forty-six hundred thirty-six.
5. Four thousand six hundred thirty-six.
6. Twenty-six hundred six.

D. It is sometimes necessary to relieve the memory by writing the numbers used, and the results obtained. This requires an acquaintance with what is called WRITTEN ARITHMETIC.

When writing large numbers for addition, we place them in a column, so that figures of the same denomination shall come under each other, *i. e.*, so that units shall come under units, tens under tens, &c. We then begin at the right hand, and add the columns separately, as in the following *examples*:—

1. What is the sum of 723 + 896 + 589 + 967?

SOLUTION. Writing the numbers as opposite, we first add the units' column: 7 units + 9 units + 6 units + 3 units = 25 units = 2 tens and 5 units. Writing 5 units, and adding 2 tens to the tens' column, we have 2 tens + 6 tens + 8 tens + 9 tens + 2 tens = 27 tens = 2 hundreds, and 7 tens. Writing 7 tens, and adding 2 hundreds to the hundreds' column, we have 2 hundreds + 9 hundreds + 5 hundreds + 8 hundreds + 7 hundreds = 31 hundreds, which, being the sum of the last column, we write. The answer, therefore, is 3175.

723
896
589
967
3175

PROOF. Examine the work carefully to see if any error can be detected. Or, add the numbers again beginning at the top of the column.

NOTE.—The design of this work renders it impracticable to give further illustrations here, but the Teacher can readily supply them if they are needed by the class. (See THE COMMON SCHOOL ARITHMETIC, or ARITHMETIC AND ITS APPLICATIONS.)

E. Add the following:—

1.	2.	3.	4.	5.
249	417	827	296	843
372	286	613	178	248
648	439	428	859	537
376	579	854	438	429

6. 425 + 487 + 569 + 837 + 694?
7. 854 + 308 + 560 + 746 + 593?
8. 672 + 481 + 326 + 425 + 519?
9. 243 + 495 + 826 + 324 + 476?
10. 627 + 756 + 434 + 874 + 999?

F. We write large numbers for Subtraction, so that figures of the same denomination shall come under each other, and subtract as in the following example:—

1. 8436 — 6122 = what number?

SOLUTION. Writing the numbers as opposite, we have 2 units from 6 units leave 4 units; 2 tens from 3 tens leave 1 ten; 1 hundred from 4 hundreds leave 3 hundreds; 6 thousands from 8 thousands leave 2 thousands. Therefore, the answer is 2 thousands, 3 hundreds, 1 ten, and 4 units, or 2314.

8436
6122
2314

In the same manner perform the following:—

2. 4893 — 1231?
3. 5987 — 3125?
4. 8958 — 6713?
5. 4867 — 1614?
6. 9318 — 2106?
7. 6985 — 1401?

G. If a figure of the subtrahend is larger than the corresponding figure of the minuend, we take one of the next higher denomination of the minuend, and reduce (*i. e.*, change) it to the required denomination, as in the following example:—

1. How much is 947 — 458?

SOLUTION. As we cannot subtract 8 units from 7 units, we reduce one of the 4 tens to its value in units: 1 ten = 10 units, which, added to the 7 units, gives 17 units; 8 units from 17 units = 9 units.

8, 13, 17 = { changed minuend.
9 4 7 = Minuend.
4 5 8 = subtrahend.
4 8 9 = remainder.

As we cannot subtract 5 tens from the 3 tens left in the minuend, we reduce one of the 9 hundreds to its value in tens: 1 hundred = 10 tens, which, added to the 3 tens = 13 tens; 5 tens from 13 tens = 8 tens; 4 hundreds from the 8 hundreds left in the minuend = 4 hundreds. Therefore, 947 — 458 = 4 hundreds, 8 tens, and 9 units, or 489.

PROOF. Add the subtrahend and remainder together; *if their sum is* equal to the minuend, the work is correct.

if not, there is an error in the subtraction or in the addition, and the work should be re-examined to detect it.

2. 4864 — 2579?
3. 8149 — 3463?
4. 2769 — 1487?
5. 2144 — 1397?
6. 8482 — 3586?
7. 4874 — 5850?
8. 5426 — 3987?
9. 9943 — 4399?
10. 9333 — 8888?
11. 4634 — 2359?
12. 9257 — 4328?
13. 8642 — 5858?

SECTION VIII.

A. 1. The method of writing numbers by figures is called the ARABIC METHOD.

2. There is a method of expressing numbers by letters, called the ROMAN METHOD. In this method the letter I stands for one, V for five, X for ten, L for fifty, C for one hundred, D for five hundred, and M for one thousand.

3. If a letter is repeated, it indicates that the number for which it stands is repeated.

ILLUSTRATIONS. I stands for one, II for two, III for three, X for ten, XX for twenty, XXX for thirty, &c.

4. If a letter representing any number stands before a letter representing a larger number, the value of the former is subtracted from the value of the latter.

ILLUSTRATIONS. IV = 1 from 5 = 4, IX = 1 from 10 = 9, XL = 10 from 50 = 40, &c.

5. If a letter representing any number stands before a letter representing a smaller number, the value of the former is to be added to the value of the latter.

ILLUSTRATIONS. VI = 5 + 1 = 6, XI = 10 + 1 = 11, XV = 10 + 5 = 15, CX = 100 + 10 = 110, &c.

B. The following table furnishes further illustrations of this method of representing numbers:—

I = 1	XI = 11	XXI = 21
II = 2	XII = 12	XXIV = 24
III = 3	XIII = 13	XXV = 25
IV = 4	XIV = 14	XXX = 30
V = 5	XV = 15	XXXIX = 39
VI = 6	XVI = 16	XLIV = 44
VII = 7	XVII = 17	LXX = 70
VIII = 8	XVIII = 18	LXXXIX = 89
IX = 9	XIX = 19	XC = 90
X = 10	XX = 20	CXXXIX = 139

SECTION IX.

TABLES OF MONEYS, WEIGHTS, AND MEASURES.

A. The money used in the United States is called UNITED STATES MONEY or FEDERAL MONEY.

TABLE.

10 mills = 1 cent.
10 cents = 1 dime.
10 dimes = 1 dollar.
10 dollars = 1 eagle.

The coins of the United States are: the cent, the three-cent piece, the half-dime, worth 5 cents; the dime, worth 10 cents; the quarter-dollar, worth 25 cents; the half-dollar, worth 50 cents; the dollar, worth 100 cents; the quarter-eagle, worth two and a half dollars; the three-dollar piece; the half-eagle, worth 5 dollars; the eagle, worth 10 dollars; the double-eagle, worth 20 dollars; and the fifty-dollar piece.

NOTE. In reciting these tables, let the pupils say "equal" *instead of* "*make* one," the phrase often used.

The character $ placed at the left of figures, shows that they represent dollars, or values in United States Money. The dollars are separated from the dimes, cents, and mills, by a dot or period called the DECIMAL POINT. The figures at the left of this point represent dollars. Of the figures at the right of this point, the first represents dimes, the second cents, the third mills, and the fourth tenths of a mill. In reading, the dimes and cents are read together as cents.

ILLUSTRATIONS. $ 8.27 = 8 dollars, 27 cents.
$15.06 = 15 dollars, 06 cents.
$2.327 = 2 dollars, 32 cents, 7 mills.

Read the following:—

1. $4.28	3. $82.36	5. $40.03
2. $5.37	4. $75.07	6. $28.79

B. ENGLISH or STERLING MONEY is the money used in England.

FULL TABLE.	ABBREVIATED TABLE.
4 farthings = 1 penny.	4 far. = 1d.
12 pence = 1 shilling.	12 d. = 1s.
20 shillings = 1 pound.	20 s. = 1£.

NOTE.—The English pound is worth *about* $4.84 (almost 5 dollars); the English shilling is worth *about* a quarter of a dollar; and the English penny is worth *about* 2 cents.

C. AVOIRDUPOIS WEIGHT is used for weighing iron, flour, sugar, wool, coal, and almost all articles except gold, silver, and jewels.

FULL TABLE.	ABBREVIATED TABLE.
16 drams = 1 ounce.	16 dr. = 1 oz.
16 ounces = 1 pound.	16 oz. = 1 lb.
25 pounds = 1 quarter.	25 lb. = 1 qr.
4 quarters = 1 hundred weight.	4 qr. = 1 cwt.
20 hundred weight = 1 ton.	20 cwt. = 1 T.

D. TROY WEIGHT is used for weighing gold, silver, jewels, &c.

FULL TABLE.	ABBREVIATED TABLE.
24 grains = 1 pennyweight.	24 gr. = 1 dwt.
20 pennyweights = 1 ounce.	20 dwt. = 1 oz.
12 ounces = 1 pound.	12 oz. = 1 lb.

NOTE. A pound Avoirdupois is heavier than a pound Troy; but an ounce Avoirdupois is not as heavy as an ounce Troy.

E. APOTHECARIES' WEIGHT is used in compounding or mixing medicines.

FULL TABLE.	ABBREVIATED TABLE.
20 grains = 1 scruple.	20 gr. = 1 ℈.
3 scruples = 1 dram.	3 ℈ = 1 ʒ.
8 drams = 1 ounce.	8 ʒ = 1 ℥.
12 ounces = 1 pound.	12 ℥ = 1 ℔.

F. LONG MEASURE is used for measuring lengths and distances.

FULL TABLE.		ABBREVIATED TABLE.	
12 lines	= 1 inch.	12 l.	= 1 in.
12 inches	= 1 foot.	12 in.	= 1 ft.
3 feet	= 1 yard.	3 ft.	= 1 yd.
5½ yards, or 16½ feet	= 1 rod, or pole.	5½ yd., or 16½ ft.	= 1 rd. or p.
40 rods	= 1 furlong.	40 rd.	= 1 fur.
8 furlongs	= 1 mile.	8 fur.	= 1 m.
3 miles	= 1 league.	3 m.	= 1 le.

G. Cloths, silks, &c., are measured by CLOTH MEASURE.

FULL TABLE.	ABBREVIATED TABLE.
2¼ inches = 1 nail.	2¼ in. = 1 na.
4 nails = 1 quarter.	4 na. = 1 qr.
4 quarters = 1 yard.	4 qr. = 1 yd.

H. SQUARE MEASURE is used in measuring land, and all kinds of surfaces.

FULL TABLE.		ABBREVIATED TABLE.	
144 square inches	= 1 square foot.	144 sq. in.	= 1 sq. ft.
9 square feet	= 1 square yard.	9 sq. ft.	= 1 sq. yd.
30¼ square yards, or 272¼ square feet,	= 1 sq. rod.	30¼ sq. yd. or 272¼ sq. ft.	= 1 sq. rd.
40 square rods	= 1 rood.	40 sq. rd.	= 1 R.
4 roods	= 1 acre.	4 R.	= 1 A.
640 acres	= 1 square mile.	640 A.	= 1 sq. m.

I. CUBIC MEASURE is used in measuring solids.

FULL TABLE.		ABBREVIATED TABLE.	
1728 cubic inches	= 1 cubic foot.	1728 cu. in.	= 1 cu. ft.
27 cubic feet	= 1 cubic yard.	27 cu. ft.	= 1 cu. yd.
16 cubic feet	= 1 cord foot.	16 cu. ft.	= 1 cd. ft.
8 cord feet, or 128 cubic feet	= 1 cord of wood.	8 cd. ft. or 128 cu. ft.	= 1 cd. wd.

J. CIRCULAR MEASURE is used for measuring angles and arcs of circles.

FULL TABLE.	ABBREVIATED TABLE.
60 seconds = 1 minute.	60″ = 1′.
60 minutes = 1 degree.	60′ = 1°.
360 degrees = 1 circumference.	360° = 1 circ.

K. DRY MEASURE is used for measuring grain, nuts, salt, &c.

FULL TABLE.	ABBREVIATED TABLE.
2 pints = 1 quart.	2 pt. = 1 qt.
8 quarts = 1 peck.	8 qt. = 1 pk.
4 pecks = 1 bushel.	4 pk. = 1 bu.

NOTE. The chaldron of 36 bushels is sometimes used in measuring coal. Ch. is the sign for chaldron.

L. LIQUID MEASURE is used for measuring all kinds of liquids.

FULL TABLE.	ABBREVIATED TABLE.
4 gills = 1 pint.	4 gil. = 1 pt.
2 pints = 1 quart.	2 pt. = 1 qt.
4 quarts = 1 gallon.	4 qt. = 1 gal.

NOTE. The hogshead of 63 gallons is used in estimating the contents of reservoirs, or other large bodies of water, but in all other cases the term hogshead is not a definite measure. Casks containing from 50 or 60, to 100 or 200 gallons, are called hogsheads.

A barrel of cider is usually reckoned at 31½ gallons.

M. TIME is reckoned in years, months, days, &c.

FULL TABLE.	ABBREVIATED TABLE.
60 seconds = 1 minute.	60 sec. = 1 min.
60 minutes = 1 hour.	60 min. = 1 h.
24 hours = 1 day.	24 h. = 1 d.
7 days = 1 week.	7 d. = 1 wk.
365¼ days, or 52 weeks, 1¼ days } = 1 year.	365¼ d. or 52 wk. 1¼ d. } = 1 y.

To avoid the inconvenience of reckoning ¼ day with each year, every fourth year (called leap year) is reckoned at 366 days, and the others at 365.

The year is divided into 12 months, which differ somewhat in length, as is seen in the following

N. TABLE OF MONTHS.

January has 31 days.	July has 31 days.
February has 28 days.*	August has 31 days.
March has 31 days.	September has 30 days
April has 30 days.	October has 31 days.
May has 31 days.	November has 30 days.
June has 30 days.	December has 31 days.

* *Except in* leap year, when it has 29.

C. MISCELLANEOUS TABLE.

12 things = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 things = 1 score.
A barrel of beef or pork weighs 200 lb.
A barrel of flour weighs 196 lb.
24 sheets of paper = 1 quire.
20 quires of paper = 1 ream.

SECTION X.

A. 1. Such questions as "How many are 3 times 2," are questions in MULTIPLICATION. We say then, that—

2. MULTIPLICATION is a process by which we ascertain how many units there are in any number of times a given number.

3. The number to be taken some number of times is called the MULTIPLICAND; the number which shows how many times the Multiplicand is to be taken is called the MULTIPLIER; and the result is called the PRODUCT.

ILLUSTRATION.—The question "How many are 3 times 2," requires us to multiply 2 by 3; 2 is the Multiplicand, 3 is the Multiplier, and the answer, 6, is the Product.

4. The Multiplier and Multiplicand are called FACTORS of the Product, and the Product is called a MULTIPLE of its factors.

ILLUSTRATION.—3 and 2 are factors of 6, because 3 times 2 = 6. For the same reason 6 is a multiple of 3, and of 2.

5. Two oblique lines crossing each other form the sign

of multiplication. In reading it may be called either "times" or "multiplied by."

ILLUSTRATION.—"8 × 2 = 6" may be read "8 times 2 = 6" or "8 multiplied by 2 = 6."

TO THE TEACHER.—It will probably be best to have the pupils at first read the sign of multiplication as though written "times," but they should eventually learn to read and use it in either form.

B.

2 × 1 or 1 × 2 = 2.
2 × 2 = 4.
3 × 2 or 2 × 3 = 6.
4 × 2 or 2 × 4 = 8.
5 × 2 or 2 × 5 = 10.
6 × 2 or 2 × 6 = 12.
7 × 2 or 2 × 7 = 14.
8 × 2 or 2 × 8 = 16.
9 × 2 or 2 × 9 = 18.
10 × 2 or 2 × 10 = 20.

3 × 3 = 9.
4 × 3 or 3 × 4 = 12
5 × 3 or 3 × 5 = 15.
6 × 3 or 3 × 6 = 18.
7 × 3 or 3 × 7 = 21.
8 × 3 or 3 × 8 = 24.
9 × 3 or 3 × 9 = 27.
10 × 3 or 3 × 10 = 30.

C.

1. * times 2 = 12?
2. * times 6 = 18?
3. * times 3 = 18?
4. * times 5 = 10?
5. 3 times * = 27?
6. 2 times * = 20?
7. 8 times * = 16?
8. 10 times * = 30?
9. 21 = * times 7?
10. 8 = * times 2?
11. 15 = * times 3?
12. 9 = * times 3?
13. 10 = 5 times *?
14. 12 = 4 times *?
15. 14 = 2 times *?
16. 27 = 9 times *?

D.

1. 5 times 2, plus 8 = * times 3?
2. 7 times 3, minus 5 = * times 8?
3. 2 times 9, plus 6 = * times 3?
4. 3 times 7, plus 9 = * times 10?
5. 4 times 3, plus 8 = * times 2?
6. 8 times 3, minus 10 = 7 times *?
7. 5 times 2, plus 2 times 7 = 3 times *?
8. 3 times 4, plus 6 times 2 = 8 times *?
9. 9 times 3, minus 3 times 2 = 3 times *?
10. *8 times 10, minus* 6 times 3 = 4 times *?

TO THE TEACHER.—The reasoning processes illustrated in the following solutions are very important, and should be thoroughly understood by the pupils. Not till by much drill and many repetitions they have become perfectly familiar, can they safely be omitted or neglected. Indeed, if the pupil must in his first exercises omit either, it is far better to omit the answer and give the reasoning process, than to omit the reasoning process and give the answer only.

E. 1. 9 qt. = how many pt.?

SOLUTION. Since 1 qt. = 2 pt., 9 qt., must equal 9 times 2 pt., which are 18 pt. Therefore, 9 qt. = 18 pt.

2. 6 qt. = * pt.?
3. 8 gal. = * qt.?
4. 2 wk. = * da.?
5. 8 dimes = * cents?
6. 9 yd. = * ft.?
7. 3 bu. = * pk.?
8. 2 pk. = * qt.?
9. 3 sq. yd. = * sq. ft.
10. 3 ℥ = * ʒ?
11. 3 wk. = * da.?
12. 2 gal. = * qt.?
13. 5 yd. = * ft.?

F. 1. How much will 3 books cost at 4 dollars each?

SOLUTION. If 1 book costs 4 dollars, 3 books will cost 3 times 4 dollars, which are 12 dollars. Therefore, 3 books at 4 dollars each will cost 12 dollars.

How much will—

2. 9 apples cost at 2 cents each?
3. 7 vests cost at 3 dollars each?
4. 3 hats cost at 4 dollars each?
5. 3 bags contain if each contains 9 quarts?
6. 8 jugs hold if each holds 3 gallons?
7. 7 trout weigh if each weighs 2 pounds?
8. 5 barrels of beef weigh if each weighs 2 hundred weight?
9. 3 pairs of boots cost at $6 per pair?
10. 10 yards of cloth cost at $3 per yard?
11. 4 pictures cost if each cost 2 cents?
12. Joseph earn in 3 weeks if he earns $6 per week?
13. George earn in 10 days if he earns $3 per day?

14. How many days will it take 1 man to do a piece of work which 4 men can do in 3 days?

15. How many days will it take 1 man to do a piece of work which 3 men can do in 9 days?

16. If 8 men can do a piece of work in 2 days, how many days will it take 1 man to do it?

17. If it takes 3 men to do a piece of work in 7 days, how many men will it take to do it in 1 day?

18. If it takes 10 men to do a piece of work in 3 days, how many men will it take to do it in 1 day?

G. **1.** 21 ft. = * yd.?

SOLUTION. Since 3 feet equal 1 yard, 21 feet must equal as many yards as there are times 3 in 21, which are 7 times. Therefore, 21 ft. = 7 yd.

2. 18 pt. = * qt.?
3. 28 da. = * wk.?
4. 24 ʒ = * ℥?
5. 16 qt. = * pk.?
6. 12 pk. = * bu.?
7. 14 pt. = * qt.?
8. 8 qt. = * gal.?
9. 27 sq. ft. = * sq. yd?
10. 30 cents = * dimes?
11. 21 da. = * wk.?
12. 14 da. = * wk.?
13. 21 ft. = * yd.?
14. 16 ʒ = * ℥?
15. 18 pt. = * qt.?

H. **1.** How many hats at $5 each can be bought for $15?

SOLUTION. Since 1 hat can be bought for 5 dollars, as many hats can be bought for 15 dollars, as there are times 5 in 15, which are 3 times. Hence, 3 hats at $5 each can be bought for $15.

2. How many caps at $2 each can be bought for $12?
3. How many sheep at $8 each can be bought for $24?
4. How many coats at $6 each can be bought for $12?
5. How many tables at $7 each can be bought for $21?
6. How many lamps at $2 each can be bought for $16?
7. ***How many chairs*** at $3 each can be bought for $27?

8. How many jugs each holding 3 qt. can be filled from 24 qt. of oil?

9. How many balls each weighing 9 lb. can be made from 27 lb. of iron?

10. How many three-quart jugs can be filled from 15 qt. of vinegar?

11. How many two-ounce bullets can be made from 20 ounces of lead?

12. How many three-peck baskets can be filled from 21 pecks of corn?

SECTION XI.

A. $4 \times 4 = 16$.
5×4, or $4 \times 5 = 20$
6×4, or $4 \times 6 = 24$.
7×4, or $4 \times 7 = 28$.
8×4, or $4 \times 8 = 32$.
9×4, or $4 \times 9 = 36$.
10×4, or $4 \times 10 = 40$

$5 \times 5 = 25$.
6×5, or $5 \times 6 = 30$.
7×5, or $5 \times 7 = 35$
8×5, or $5 \times 8 = 40$.
9×5, or $5 \times 9 = 45$.
10×5, or $5 \times 10 = 50$.

B. 1. $*$ times $7 = 28$?
2. $*$ times $5 = 40$?
3. $*$ times $9 = 36$?
4. $*$ times $6 = 30$?
5. $40 =$ $*$ times 4?
6. $25 =$ $*$ times 5?
7. $32 =$ $*$ times 8?
8. $32 =$ $*$ times 4?
9. 5 times $* = 45$?
10. 6 times $* = 24$?
11. 9 times $* = 36$?
12. 4 times $* = 32$?
13. $28 = 7$ times $*$?
14. $35 = 5$ times $*$?
15. $40 = 4$ times $*$?
16. $24 = 6$ times $*$?

C. 1. Such questions as "28 equals how many times 7?" "$40 = 4$ times what number?" &c., are questions in DIVISION.

2. DIVISION is a process by which we ascertain how many *times one* given number is contained in another, or

by which we ascertain what number is contained a given number of times in another given number.

3. The number to be divided is called the DIVIDEND, the number by which we divide is called the DIVISOR, and the result is called the QUOTIENT.

ILLUSTRATION. In "24 = * times 6?" 24 is to be divided by 6. 24 is the dividend, 6 is the divisor, and the answer 4 is the quotient.

4. A single horizontal line, with a dot above it and a dot below it, forms the sign of division. It shows that the number before it is to be divided by the number after it.

ILLUSTRATION. "12 ÷ 2 = 6" means the same as "The quotient of 12 divided by 2 = 6."

5. The comma, when used in connection with Arithmetical signs, shows that the *result* of all the preceding operations is to be considered in connection with the sign following it.

ILLUSTRATION. "6 × 7 + 21, ÷ 9," means that the sum of 6 times 7 plus 21 is to be divided by 9, or that 68 is to be divided by 9.

D. What is the quotient of—

1. 15 ÷ 3?	5. 40 ÷ 5?	9. 28 ÷ 7?
2. 36 ÷ 9?	6. 32 ÷ 4?	10. 35 ÷ 5?
3. 24 ÷ 8?	7. 25 ÷ 5?	11. 45 ÷ 9?
4. 14 ÷ 7?	8. 20 ÷ 2?	12. 21 ÷ 3?

E. 1. 7 × 5, + 5, ÷ 4, + 22, = * times 8?

SOLUTION. 7 times 5 are 35, plus 5 are 40, divided by 4 are 10, plus 22 are 32, which equals 4 times 8.

2. 9 × 3, + 9, ÷ 4, × 2, = * times 3?
3. 8 × 2, + 4, ÷ 2, + 8, + 6, = * times 8?
4. 10 × 4, — 12, ÷ 4, + 20, = * times 9?
5. 30 + 20, ÷ 10, × 7, — 3, = * times 4?
6. 5 + 9 + 10 + 6 — 9, ÷ 3, + 20 = * times 4?

F. 1. 4 yd. 2 ft. = how many feet?

SOLUTION. Since 1 yd. = 3 ft., 4 yd. must equal 4 times 3 ft., which are 12 ft., and 2 ft. added are 14 ft. Therefore, 4 yd. 2 ft. = 14 ft.

2. 3 wk. 6 da. = * da.?
3. 5 pk. 4 qt. = * qt.?
4. 9 yd. 2 ft. = * ft.?
5. 5 sq. yd. 6 sq. ft. = * sq. ft.?
6. 4 m. 7 fur. = * fur.?
7. 8 A. 3 R. = * R.?

G. 1. 6 gal. 3 qt. = how many times 2 gal. 1 qt.?

ABBREVIATED SOLUTION. 6 gal. 3 qt. = 27 qt.; 2 gal. 1 qt. = 9 qt.; and 27 qt. contains 9 qt. as many times as 27 contains 9, which is 3 times. Hence, 6 gal. 3 qt. = 3 times 2 gal. 1 qt.

2. 8 bu. 3 pk. = * times 1 bu. 1 pk.?
3. 4 m. 4 fur. = * times 1 m. 1 fur.?
4. 3 ℥ 6 ʒ = * times 1 ℥ 2 ʒ?
5. 3 wk. 3 da. = * times 4 da.?
6. 7 gal. 2 qt. = * times 3 qt.?
7. 2 wk. 4 da. = * times 6 da.?
8. 6 ʒ 2 ℈ = * times 2 ℈?
9. 8 yd. = * times 1 yd. 1 ft.?
10. 4 gal. 2 qt. = * times 1 gal. 2 qt.

H. 1. Edward bought 7 oranges at 4 cents each, and then had 12 cents left. How much money did he have before he bought the oranges?

SOLUTION. We must first find what the oranges cost. If 1 orange cost 4 cents, 7 oranges must have cost 7 times 4 cents, which are 28 cents; and if he paid 28 cents for oranges, and had 12 cents left, he must have had in the first place 28 cents plus 12 cents, which are 40 cents. Hence, he had 40 cents before he bought the oranges.

2. Francis says that he has money enough to buy 4 cocoa nuts at 9 cents apiece, and still have 5 cents left. How much money has he?

3. William has 9 three-cent pieces, and 8 cents besides. How many cents has he in all?

4. If Augustus has 37 apples, how many will he have left after giving 4 of his companions 7 apples apiece?

5. Sarah bought 9 spools of thread at 5 cents apiece, and then had money enough left to buy 2 skeins of silk at 3 cents per skein. How much money had she at first?

6. Arthur has 4 half-dimes, 2 three-cent pieces, and 11 cents. How much money has he?

7. Richard bought 8 newspapers at 2 cents apiece, and sold them for 4 cents apiece. How much did he gain on them?

8. If Daniel has 50 chestnuts, how many will he have left after giving 4 of his companions 9 chestnuts apiece?

9. I bought 9 yards of cloth at $4 per yard, but was obliged to sell it for $12 less than cost. For how much did I sell it?

10. Willie exchanged 27 cents for their value in three-cent pieces. How many three-cent pieces did he get?

11. Amelia had 38 apples. She ate 2, and divided the rest among her playmates, giving 4 to each. Among how many did she divide them?

12. Simon had 42 cents. He gave 10 cents for a writing book, and 5 for an inkstand, and exchanged the rest of his money for three-cent pieces. How many three-cent pieces did he get?

13. How many pen-holders, at 3 cents apiece, can be bought for 15 cents?

14. How many books at 10 cents each can be bought for *8 half-dimes?*

15. How many oranges at 4 cents each can be bought for 6 three-cent pieces and 2 cents?

16. How many apples at 3 cents apiece can be bought for 6 oranges at 4 cents each?

17. How many pairs of boots at $5 a pair, can be bought for 10 yards of cloth at $3 per yard?

18. A man put 10 quarts of berries into boxes each holding 2 qt. 1 pt. How many boxes did he fill?

19. How many five-peck baskets will be required to hold 8 bu. 3 pk. of corn?

20. Rufus had a string 5 yards 1 foot long, which he cut into pieces 2 feet long. How many pieces did it make?

21. A newsboy sold 9 papers at 3 cents apiece, and after spending 9 cents, gave the rest of the money for papers at 2 cents apiece. How many papers did he get?

22. A man gave 8 hats at $4 each and $8 in money, for coats at $10 each. How many coats did he receive?

23. A man put 7 gal. 2 qt. of molasses into jugs each holding 3 qt. How many jugs did he fill?

24. An apothecary put 5 ʒ 1 ℈ of powders into papers, each holding 2 ℈. How many papers did he fill?

25. If I should pay 18 cents for apples at 2 cents each, and should sell them for 3 cents each, how many apples should I buy, and how many cents should I sell them for?

26. I paid $24 for cloth at $3 per yard, and sold it for $5 per yard. How many dollars did I receive for it?

27. I exchanged 27 silver dollars for three-dollar bills, and then exchanged the bills for oats, receiving 9 bushels for each bill. How many bushels of oats did I obtain?

28. If I should exchange 18 cents for their value in three-cent pieces, and then should get 5 apples for each three-cent piece, how many apples should I get?

SECTION XII.

A. $6 \times 6 = 36$. $7 \times 7 = 49$.
7×6 or $6 \times 7 = 42$. 8×7 or $7 \times 8 = 56$.
8×6 or $6 \times 8 = 48$. 9×7 or $7 \times 9 = 63$.
9×6 or $6 \times 9 = 54$. 10×7 or $7 \times 10 = 70$.
10×6 or $6 \times 10 = 60$.

$8 \times 8 = 64$. $9 \times 9 = 81$.
9×8 or $8 \times 9 = 72$. 10×9 or $9 \times 10 = 90$.
10×8 or $8 \times 10 = 80$. $10 \times 10 = 100$.

B.
1. $* \times 6 = 36$?
2. $* \times 8 = 48$?
3. $* \times 7 = 49$?
4. $* \times 9 = 72$?
5. $8 \times * = 56$?
6. $6 \times * = 54$?
7. $9 \times * = 81$?
8. $7 \times * = 63$?
9. $42 = * \times 7$?
10. $54 = * \times 9$?
11. $64 = * \times 8$?
12. $72 = * \times 9$?
13. $6 \times * = 36$?
14. $7 \times * = 56$?
15. $9 \times * = 90$?
16. $5 \times * = 45$?

C. What is the quotient of—

1. $81 \div 9$?
2. $36 \div 4$?
3. $48 \div 8$?
4. $72 \div 9$?
5. $56 \div 8$?
6. $21 \div 7$?
7. $45 \div 5$?
8. $63 \div 9$?
9. $100 \div 10$?
10. $24 \div 8$?
11. $72 \div 8$?
12. $63 \div 7$?

D.
1. $7 \times 8, + 7, \div 7, \times 4, = *$ times 6?
2. $8 \times 10, - 8, \div 9, + 1, \times 6, - 5, = *$ times 7?
3. $6 \times 8, - 12, \div 4, \times 3, + 10$ times $4, - 3, = *$ times
4. $7 \times 6, + 3, \div 9, \times 5, + 8, \div 7, \times 5, - 2, = *$ times
5. $8 \times 8, + 4$ times 9, plus 21, $\div 9, \times 2, + 7, \div 5$?
6. $9 \times 10, - 18, \div 9, \times 6, + 5$ times $3, = *$ times 9

E. 1. How many are 9 times 4 yd. 2 ft.?

Solution. 9 times 4 yd. = 36 yd.; 9 times 2 ft. =

18 ft.; and since 3 ft. equal 1 yd., 18 ft. must equal as many yards as 3 is contained times in 18, which are 6 times Hence, 18 ft. = 6 yd., which added to 36 yd. = 42 yd. Hence, 9 times 4 yd. 2 ft. = 42 yd.

2. 4 times 7 gal. 3 qt.?
3. 7 times 8 wk. 4 da.?
4. 6 times 9 yd. 2 ft.?
5. 8 times 7 cwt. 3 qr.?
6. 9 times 5 ʒ 2 ℈?
7. 9 times 8 sq. yd. 4 sq. ft.
8. 6 times 7 bu. 6 pk.?
9. 6 times 8 gal. 2 qt.?
10. 3 times 5 sq. yd. 6 sq. ft.
11. 8 times 9 m. 7 fur.?

F. 1. 5 wk. 1 da. = * times 1 wk. 2 da.?

ABBREVIATED SOLUTION. 5 wk. 1 da. — 36 days; 1 wk. 2 da. = 9 days; and 36 da. contains 9 days as many times as 36 contains 9, which is 4 times. Hence, 5 wk. 1 da. = 4 times 1 wk. 2 da.

2. 3 pk. 6 qt. = * times 1 pk. 2 qt.?
3. 6 yd. 2 ft. = * times 1 yd. 2 ft.?
4. 9 gal. = * times 2 gal. 1 qt.?
5. 6 wk. 3 da. = * times 1 wk. 2 da.?
6. 3 ʒ 6 ʒ = * times 1 ʒ 2 ʒ?
7. 3 sq. yd. 5 sq. ft. = * times 8 sq. ft.?
8. 8 cwt. 3 qr. = * times 1 cwt. 3 qr.?
9. 7 pk. 4 qt. = * times 1 pk. 2 qt.?
10. 7 wk. 5 da. = * times 1 wk. 2 da.?

G. 1. I sold 8 quarts of cherries at 6 cents per quart, and one quart for 8 cents. How many cents did I receive for them? How many pounds of rice at 7 cents per pound could be bought with the money thus received?

2. If George walks at the rate of 15 rods per minute, and William walks at the rate of 21 rods per minute, how many more rods per minute does William walk than George? How many more rods in 9 minutes?

3. If Susan gains 9 merit-marks per day, and loses 2 per day, how many will she have at the end of 8 days?

4. How many dimes will be received for 9 pecks of chestnuts at one dime per quart?

5. I sold 6 quarts of blackberries at 10 cents per quart, receiving in payment 5 three-cent pieces, and the rest in half-dimes. How many half-dimes did I receive?

6. How much more will 8 oranges cost at 6 cents each, than 9 oranges at 5 cents each?

7. How many bags, each containing 1 bu. 1 pk., can be filled from 6 bu. 1 pk. of meal?

8. How many house-lots, each containing 1 A. 2 R., can be made from 10 A. 2 R. of land?

9. How many pictures, at 2d. 1 qr., each can be purchased for 6d. 3 qr.?

10. How many bushels are there in 8 bags, each containing 3 pecks?

11. A furniture-dealer gave 6 bureaus worth 7 dollars each, and 3 dollars in money, for chairs at 9 dollars per dozen. How many dozen chairs did he buy?

12. A fur-dealer gave 8 caps at 5 dollars apiece, and 2 dollars in money, for muffs at 6 dollars apiece. How many muffs did he receive?

13. Bought 6 bags, each containing 3 bu. 2 pk. of peanuts, and put them into casks each holding 3 bushels. How many casks did they fill?

14. Austin had a basket which held just 3 pk. 4 qt., and another which held just 5 pecks. He gathered nuts enough to fill the smallest basket 10 times. How many times would they fill the large basket?

15. A lady bought sheeting enough to make 8 sheets, each containing 5 yd. 1 qr., but afterwards concluded to put 6 yards in a sheet. How many sheets could she make?

16. By buying a lot of wood at $4 per cord, and selling

it at $6 per cord, I gained $18. How many dollars did I gain on each cord? How many cords did I buy? How many dollars did I pay for the entire lot?

17. By buying flour at $5 per barrel, and selling it for $8 per barrel, I gained $24. How many barrels did I buy, and what did I pay for the lot?

18. By buying cloth at $5 per yard, and selling it at $3 per yard, I lost $20. How many dollars did I pay for it?

19. A man bought a lot of coal at $4 per ton, and sold it for $8 per ton, by which he gained $36. How many dollars did he pay for it?

20. A boy earned 12 cents by doing some errands, and invested the money in papers at 2 cents each. He sold the papers at 4 cents each, and expended the money thus received for papers at 3 cents each. He sold 6 of the papers for 5 cents each, and the rest for 2 cents each. He then spent 4 cents, and gave the rest of his money for Havana oranges at 6 cents apiece. He gave 1 of the oranges to his mother, and sold the rest at 8 cents apiece. How much did he receive for them?

21. A laborer worked 6 weeks for 9 dollars per week, and putting 6 dollars with the money thus earned, he bought coal at 6 dollars per ton. How many tons did he buy? After laying aside 2 tons for his own use, he sold the remainder for 7 dollars per ton, receiving in payment 2 dollars in money, and the rest in flour at 9 dollars per barrel. How many barrels of flour did he receive?

22. Joseph travels 3 miles an hour, and William travels 5 miles an hour. If they start at the same place at the same time and travel in opposite directions, how far apart will they be in 9 hours?

23. What would have been the answer to the last question if they had travelled in the same direction?

SECTION XIII.

A. 1. 29 = * times 6?

FIRST SOLUTION. 29 = 4 times 6 with 5 remaining, for 4 times 6 = 24, and 24 + 5 = 29.

SECOND SOLUTION. 29 = 24 + 5, and 24 = 4 times 6. Hence, 29 = 4 times 6 with 5 remainder.

NOTE. The remainder is really an undivided part of the dividend, and might be subtracted from it without affecting the quotient. In reality, but a part of the dividend is divided.

2. 46 = * times 5?
3. 31 = * times 9?
4. 57 = * times 7?
5. 88 = * times 9?
6. 61 = * times 8?
7. 37 = * times 7?
8. 53 = * times 6?
9. 27 = * times 8?
10. 52 = * times 9?
11. 48 = * times 7?
12. 69 = * times 9?
13. 23 = * times 11?

B. What is the quotient of—

1. 42 ÷ 4?
2. 83 ÷ 9?
3. 75 ÷ 8?
4. 24 ÷ 7?
5. 75 ÷ 9?
6. 46 ÷ 5?
7. 23 ÷ 2?
8. 19 ÷ 8?
9. 47 ÷ 7?
10. 39 ÷ 6?
11. 51 ÷ 8?
12. 54 ÷ 7?
13. 97 ÷ 9?
14. 48 ÷ 7?
15. 43 ÷ 9?
16. 43 ÷ 4?
17. 47 ÷ 8?
18. 47 ÷ 6?
19. 63 ÷ 8?
20. 27 ÷ 5?
21. 80 ÷ 9?

C. 1. 5 × 9, ÷ 8?
2. 4 × 6, ÷ 7?
3. 5 × 9, + 8, ÷ 6?
4. 8 × 8, − 13, ÷ 9?
5. 7 × 6, + 13, ÷ 7?
6. 9 × 8, − 16, ÷ 9?
7. 9 × 6, + 10, = * × 8?
8. 4 × 9, + 17, = * × 6?
9. 7 × 7, + 13, = * × 10?
10. 9 × 9, − 43, = * × 4?
11. 4 × 10, + 23, = * × 8?
12. 6 × 9, − 19, = * × 6?

D. 1. 41 days = how many weeks?

SOLUTION. Since 7 days = 1 wk., 41 da. must equal as many weeks as there are times 7 in 41, which are 5 times with 6 remainder. Hence, 41 da. = 5 wk. 6 da.

2. 83 qr. = * yd.?
3. 87 ʒ = * ℥?
4. 70 qt. = * pk.?
5. 19 ft. = * yd.?
6. 23 ft. = * yd.?
7. 46 fur = * m.?
8. 59 da. = * wk.?
9. 37 cd. ft. = * cd.?
10. 60 sq ft. = * sq. yd.?
11. 20 m. = * le.?

E. 1. If Moses has 35 cents, how many pencils at 6 cents each can he buy with his money?

SOLUTION. If he can buy 1 pencil for 6 cents, he can buy as many pencils for 35 cents as there are times 6 in 35, which are 5 times and 5 remainder. Therefore, if Moses has 35 cents, he can buy 5 pencils at 6 cents each, and have 5 cents remaining.

How many—

2. Sheep at $9 each can be bought for $85?
3. Ploughs at $6 each can be bought for $58?
4. Shawls at $7 each can be bought for $39?
5. Books at $3 each can be bought for $29?
6. Terms tuition at $7 per term will $27 pay for?
7. Hats at $4 each can be bought for $39?
8. Five-quart jugs can be filled from 9 gal. 3 qt. of milk?
9. Coats at $7 each can be bought for $50?
10. Casks each holding 2 bu. 1 pk. can be filled from 10 bu. 3 pk. of oats?
11. Bags each holding 1 bu. 3 pk. can be filled from 9 bu. 1 pk. of grain?

12. A person who owed $39, paid as much as possible in five-dollar bills, and the rest in one-dollar bills. How many bills of each kind did he pay?

$9\frac{35}{5} - 8$

13. William had 35 cents. He bought as many writing-books at 9 cents each as he could pay for, and spent the rest of his money for pens at 2 cents apiece. How many writing-books did he buy? How many pens?

14. A tailor paid $35 for silk velvet at $5 per yard, and made it into vests, putting 3 quarters into each vest. How many vests did he make?

15. One "Fourth of July" Thomas had 29 cents. He bought as many bunches of crackers at 10 cents per bunch, as he could pay for, and then spent the rest of his money for cherries, at the rate of 7 for a cent. How many bunches of crackers did he buy? How many cherries?

16. Lyman has 28 cents, Horace has 50, Chester has 63, and Isaac has 47. Each bought as many pencils at 8 cents apiece as he could pay for, and gave the rest of his money to a poor woman. How many pencils did each buy, and how many cents had each to give the poor woman?

17. Robert has 6 cents, and John has 5 cents more than 8 times as many as Robert. How many half-dimes can John obtain for his money?

18. Luther has 7 marbles, and Joshua has 15 less than 6 times as many as Luther. How many times 9 marbles have the two boys together?

19. Otis has 9 cents, Ambrose has 12 cents less than 8 times as many as Otis, and if 4 times Otis's money be subtracted from Ambrose's the remainder will equal Martin's. How many cents has each of the boys? How many pencils at 6 cents apiece could each boy buy?

20. Mary had 25 books, Anna had 17 books less than Mary, and Jane had 16 books less than 9 times as many as Anna. Jane arranged her books 9 on a shelf. How many shelves did she fill?

SECTION XIV.

A. 1. 2×3 tens, or 2×30?
2. 4×2 tens, or 4×20?
3. 7×3 tens, or 7×30?
4. 8×4 tens, or 8×40?
5. 7×9 tens, or 7×90?
6. 6×5 tens, or 6×50?

B.

1. 4×3?	7. 5×40?	13. 7×80?
2. 4×30?	8. 9×30?	14. 90×9?
3. 40×3?	9. 6×40?	15. 50×6?
4. 7×9?	10. 6×80?	16. 9×60?
5. 7×90?	11. 60×8?	17. 8×90?
6. 70×9?	12. 9×70?	18. 7×70?

C. 1. How many are 6×47?

SOLUTION. 6 times 47 = 6 times 40, plus 6 times 7; 6 times 40 = 240; 6 times 7 = 42, which, added to 240 = 282. Therefore, 6 times 47 = 282.

NOTE. A little practice will enable the pupil to solve such problems by naming only the partial products and the answer, thus: 6 times 47 = 240 + 42 = 282. He should not consider them mastered till he can give the final product promptly without any oral solution.

How many are—

2. 7×96?	10. 4×27?	18. 8×37?
3. 8×84?	11. 9×82?	19. 6×28?
4. 9×87?	12. 5×97?	20. 5×43?
5. 6×94?	13. 4×23?	21. 9×81?
6. 8×23?	14. 7×94?	22. 7×73?
7. 4×29?	15. 6×43?	23. 8×81?
8. 6×54?	16. 9×84?	24. 7×56?
9. 8×76?	17. 5×69?	25. 5×88?

D. 1. How many are 6 times 498?

SOLUTION. 6 times 498 = 6 times 400, plus 6 times 90, plus 6 times 8; 6 times 400 = 2400; 6 times 90 = 540, which, added to 2400 = 2940; 6 times 8 = 48, which, added to 2940 = 2988.

2. How many are 7 times $23.38?

ABBREVIATED SOLUTION. 7 times $23.38 = 7 times $23, plus 7 times 38 cents, = $161 + $2.66, = $163.66.

How many are—

3. 8 times 527?
4. 3 times 256?
5. 4 times 531?
6. 7 times 897?
7. 9 times 326?
8. 8 times 456?
9. 5 times 378?
10. 4 times 928?
11. 8 times 746?
12. 3 times 924?
13. 7 times 692?
14. 5 times 429?
15. 6 times $2.75?
16. 4 times $8.76?
17. 9 times $32.75?
18. 8 times $27.84?
19. 5 times $97.83?
20. 4 times $28.59?
21. 7 times $31.28?
22. 9 times $43.68?
23. 7 times $32.48?
24. 5 times $84.21?
25. 9 times $27.63?
26. 8 times $53.25?

NOTE. The exercises in Written Arithmetic may be omitted if the Teacher deems it advisable.

E. In Written Arithmetic we perform examples in multiplication by the following forms of solution:—

SOLUTION TO THE FIRST EXAMPLE UNDER C.— 6 times 7 units = 42 units = 4 tens and 2 units. Writing 2 as the units' figure of the product, we add the 4 tens to the product of the tens, thus: 6 times 4 tens = 24 tens, to which, adding the 4 tens from the former product, gives 28 tens, which being the *last product* we write. The answer, then, is 282.

47
6
282

SOLUTION TO THE FIRST EXAMPLE UNDER D.—6 times 8 units = 48 units = 4 tens and 8 units. Writing 8 as the units' figure of the product, we add the 4 tens to the product of the tens, thus: 6 times 9 tens = 54 tens, and 4 tens added, equal 58 tens = 5 hundreds and 8 tens. Writing 8 as the tens' figure of the product, we add the 5 hundreds to the product of the hundreds' column, thus: 6 times 4 hundreds = 24 hundreds, and 5 hundreds added = 29 hundreds = 2 thousands and 9 hundreds, which being the last product we write. The answer, then, is 2988.

498
6
2988

NOTE. When the reductions are fully mastered, abbreviated forms like the following may be introduced with advantage.

6 times 8 = 48. Write 8 and add 4 to the next product. 6 times 9 are 54 and 4 are 58. Write 8 and add 5 to the next product. 6 times 4 are 24 and 5 are 29, which we write. Hence, 6 times 498 = 2988.

Perform in this way the examples under letters D and E.

NOTE. It will be seen that when we DO NOT write the work, we begin at the *left* hand to multiply; and when we DO write it we begin at the *right* hand. Though not absolutely necessary to observe this difference, it will as a general thing be found most convenient to do so.

F. From the preceding exercises we may infer that 40 times 3 = 4 times 30; that 50 times 27 = 5 times 270; that 80 times 273 = 8 times 2730, &c.

1. What is the product of 70 times 389?

SOLUTION. 70 times 389 = 7 times 3890, which may be found by the method before explained. Thus: 7 times 0 units = 0 units; 7 times 9 tens = 63 tens = 6 hundreds and 3 tens, &c., &c.

389
70
27230

What is the product of—

2. 20 times 64?
3. 80 times 29?
4. 40 times 36?
5. 60 times 94?
6. 90 times 37?
7. 20 times 93?
8. 30 times 84?
9. 90 times 72?
10. 30 times 979?
11. 40 times 832?
12. 70 times 687?
13. 20 times 443?
14. 60 times 927?
15. 80 times 423?
16. 50 times 975?
17. 30 times 476?

G. Since $24 = 20 + 4$, 24 times any number must equal 20 times that number plus 4 times that number. Since $86 = 80 + 6$, 86 times any number must equal 80 times that number plus 6 times that number, &c., &c.

1. What is the product of 29 times 863?

SOLUTION. Since $29 = 20 + 9$, 29 times 863 must equal 20 times 863, plus 9 times 863. We first multiply by 9, and then by 20, by the methods before explained, and add the products together as in the written work at the right.

$$\begin{array}{r} 863 \\ 29 \\ \hline 7767 \\ 17260 \\ \hline 25027 \end{array}$$

What is the product of—

2. 38 times 481?
3. 27 times 936?
4. 68 times 427?
5. 43 times 268?
6. 31 times 492?
7. 68 times 946?
8. 79 times 368?
9. 42 times 427?
10. 54 times 329?
11. 61 times 428?
12. 89×2796?
13. 38×9582?
14. 22×4858?
15. 56×9375?
16. 46×2401?
17. 63×2485?
18. 81×3258?
19. 69×2846?
20. 44×8132?
21. 74×9123?

SECTION XV.

A. What is the quotient of—

1. 12 ÷ 4?	7. 630 ÷ 9?	13. 420 ÷ 60?
2. 120 ÷ 4?	8. 720 ÷ 8?	14. 90 ÷ 30?
3. 120 ÷ 40?	9. 560 ÷ 7?	15. 270 ÷ 30?
4. 64 ÷ 8?	10. 250 ÷ 5?	16. 420 ÷ 6?
5. 640 ÷ 8?	11. 240 ÷ 3?	17. 810 ÷ 9?
6. 640 ÷ 80?	12. 720 ÷ 80?	18. 810 ÷ 90?

B. 1. What is the quotient of 476 ÷ 7?

SOLUTION. 7 is contained in 47 tens, 6 tens times, with 5 tens remaining. 5 tens equal 50, units and 6 units added, are 56 units, 7 is contained in 56 units 8 units times. Hence, the quotient of 476 ÷ 7 = 60 + 8 = 68.

NOTE. After the reductions are fully understood, the pupil should be allowed to abbreviate this explanation, thus: "7 is contained in 47 tens 6 tens times, with 5 tens remainder. 7 is contained in 56 units, 8 units times." Hence, the quotient is 68.

What is the quotient of—

2. 315 ÷ 3?	6. 672 ÷ 8?	10. 429 ÷ 8?
3. 216 ÷ 4?	7. 144 ÷ 6?	11. 413 ÷ 5?
4. 392 ÷ 8?	8. 279 ÷ 9?	12. 673 ÷ 7?
5. 217 ÷ 7?	9. 137 ÷ 2?	13. 528 ÷ 9?

C. Examples in division are performed and explained in the same manner when we write the work as when we do not. The work of the first example, letter B, would usually be written as in the annexed model:

$$7\overline{)476}\quad 68$$

D. 1. What is the quotient of 2738 ÷ 8?

SOLUTION. 8 is contained in 27 hundreds, 3 hundreds times with 3 hundreds remaining. We therefore write 3 as the hundreds' figure of the quotient. The 3 hundreds remaining = 30 tens, and 3 tens added = 33 tens. 8 is contained in 33 tens, 4 tens times, with 1 ten remaining. We therefore write 4 as the tens' figure of the quotient. The 1 ten remaining = 10 units, and 8 units added = 18 units. 8 is contained in 18 units 2 units' times and 2 units remaining. Therefore, 2738 ÷ 8 = 3 hundreds, 4 tens, and 2 units, or 342 with a remainder of 2.

$$8)2738-2$$
$$342$$

NOTE. The remainder is written after the quotient with the sign of subtraction, to show that it is an undivided part of the dividend.

2.	4756 ÷ 4?	11.	2137 ÷ 5?
3.	3297 ÷ 6?	12.	4264 ÷ 13?
4.	4347 ÷ 9?	13.	8375 ÷ 12?
5.	2981 ÷ 11?	14.	2986 ÷ 4?
6.	3297 ÷ 6?	15.	3176 ÷ 8?
7.	4361 ÷ 5?	16.	4327 ÷ 9?
8.	2459 ÷ 8?	17.	2052 ÷ 3?
9.	4272 ÷ 12?	18.	1379 ÷ 2?
10.	8943 ÷ 9?	19.	7436 ÷ 8?

E. When the divisor is a large number, it is often convenient or necessary to use the nearest number of tens, hundreds, or thousands, as a *trial divisor*, to determine the *probable* quotient figure.

ILLUSTRATIONS. In dividing by 31, 32, 33, or 34, we may make 30 or 3 the trial divisor. In dividing by 36, 37, 38, or 39, we may make 40 or 4 the trial divisor. In dividing by 35, we may make either 30 or 40 the trial divisor.

F. 1. What is the quotient of 178 ÷ 53?

SOLUTION. We may make 50 or 5 the trial divisor, for 53 is contained in 178 about the same number of times that 50 is; or, that 5 is contained in 17, which is 3 times. To test the correctness of this conclusion, we must find 3 times 53. It is 159, which, subtracted from 178, leaves 19, thus showing that 178 ÷ 53 = 3 with 19 remainder.

NOTE. In Written Arithmetic the divisor is usually placed at the left of the dividend, the quotient at the right; the product is placed beneath the dividend, and the remainder beneath the product, as in the annexed model.

53)178(3
159 = 3 × 53
19—Rem.

NOTE. The Teacher should illustrate and explain the method of proceeding when the above process gives a trial quotient figure either too large or too small.

2. 96 ÷ 24?
3. 127 ÷ 31?
4. 228 ÷ 64?
5. 583 ÷ 82?
6. 281 ÷ 29?
7. 469 ÷ 48?
8. 356 ÷ 61?
9. 429 ÷ 57?
10. 256 ÷ 38?
11. 124 ÷ 19?
12. 387 ÷ 45?
13. 621 ÷ 84?
14. 438 ÷ 62?
15. 279 ÷ 94?
16. 349 ÷ 82?
17. 624 ÷ 79?

C. 1. What is the quotient of 2856 ÷ 59.

EXPLANATION. 59 is so near 60, that we make 6 the trial divisor. Since 6 is contained 4 times in 28, we make 4 the first figure of the quotient, and infer that 59 is contained 4 tens times in 285 tens. Multiplying 59 by 4 tens, gives 236 tens, which, subtracted from 285 tens leaves 49 tens. 49 tens = 490 units, to which, adding the 6 units, gives 496 units. Since 6 is contained 8 times in 49, we make 8 the next figure of the quotient, and infer that 59

59)2856(48
236
496
472
24 = Rem.

is contained 8 times in 496. Multiplying 59 by 8 units, gives 472 units, which, subtracted from 496 units, leaves 24 units. Hence, $2856 \div 59 = 48$ with 24 remainder.

PROOF.—48 times 59, plus 24, equals 2856.

What is the quotient of—

2. $843 \div 31$?
3. $579 \div 43$?
4. $827 \div 15$?
5. $1748 \div 42$?
6. $3947 \div 49$?
7. $8246 \div 91$?
8. $4217 \div 88$?
9. $8321 \div 94$?
10. $6735 \div 83$?
11. $2317 \div 38$?
12. $7685 \div 82$?
13. $1749 \div 22$?
14. $2175 \div 25$?
15. $4802 \div 49$?
16. $6237 \div 74$?
17. $4238 \div 52$?
18. $5947 \div 75$?
19. $3286 \div 47$?

SECTION XVI.

A. 1. The multiplier and multiplicand are called FACTORS of their product. Hence,

2. A FACTOR of any given number is such an entire number as taken an entire number of times will produce the given number, or

3. FACTORS of any number are such entire numbers as multiplied together, will give that number for a product.

ILLUSTRATIONS. 5 and 3 are factors of 15, because $15 = 5 \times 3$. Again, 2 is a factor of 4, 6, 8, &c.; 3 is a factor of 6, 12, 15, &c.

4. A factor is COMMON TO TWO OR MORE NUMBERS when it is a factor of each of them.

ILLUSTRATION. 3 is a common factor of 3, 6, 9, and 12.

5. A PRIME NUMBER is a number which has no factors except itself and 1.

ILLUSTRATIONS. 1, 2, 3, 5, 7, and 11, are each prime numbers.

6. A COMPOSITE NUMBER is a number which has other factors besides itself and 1.

ILLUSTRATIONS. 4, 6, 8, and 9, are composite numbers, for $4 = 2 \times 2$, $6 = 3 \times 2$, $8 = 2 \times 4 = 2 \times 2 \times 2$, and $9 = 3 \times 3$.

7. Two numbers are PRIME TO EACH OTHER when they have no common factor except 1.

ILLUSTRATION. 4 and 9 are prime to each other; but 4 and 8 are not, because they have the common factor 2.

8. A number is DIVIDED INTO FACTORS, when any factors which will produce it are found.

ILLUSTRATIONS. In "$12 = 6 \times 2$," 12 is divided into the factors 6 and 2, but in "$12 = 2 \times 2 \times 3$," it is divided into the factors 2, 2, and 3.

9. A number is divided into its PRIME FACTORS when it is divided into factors which are all prime numbers.

ILLUSTRATIONS. $18 = 2 \times 3 \times 3$. $30 = 2 \times 3 \times 5$.

10. The product of a number taken any number of times as a factor, is called a POWER of that number.

ILLUSTRATIONS. 8, which is the product of $2 \times 2 \times 2$, *i. e.*, of 2 taken 3 times as a factor, is the *third power of two;* 25, which is the product of 5×5, *i. e.*, of 5 taken 2 times as a factor is the *second power of five.*

11. We may indicate the power of a number by writing a small figure, called an EXPONENT, above it and a little to the right.

ILLUSTRATIONS. $3^3 = 3 \times 3 \times 3$, or 3 to the third power. $2^5 = 2 \times 2 \times 2 \times 2 \times 2$, or 2 to the fifth power.

The second power of a number is sometimes called its
E, and the third power its CUBE.

STRATION. 8, or 2^3, is the *cube* of 2; 25, or 5^2, is the
of 5.

Give the prime factors of the numbers from 1 to 100,
he following model :—

1 is Prime.
2 is Prime.
3 is Prime.
$4 = 2 \times 2 = 2^2$.
5 is Prime.
$6 = 2 \times 3$.
7 is Prime.
$8 = 2 \times 2 \times 2$ or 2^3.
$9 = 3 \times 3$ or 3^2.
$10 = 2 \times 5$.

. It will also be a good exercise to write out the prime
required above. The pupils should practice on such exer-
ill the factors of each number from 1 to 100 are perfectly
r.

. What prime factors are common to 24, 36, and 48?

UTION. Dividing each number into its prime factors,
re—

$$24 = 2^3 \times 3.$$
$$36 = 2^2 \times 3^2.$$
$$48 = 2^4 \times 3.$$

s appears that 2^2 and 3 are factors of each number,
at there is no other common factor. Hence, 2, 2, and
^{2} and 3 are the prime factors required

at prime factors are common—

2. To 12 and 18?
3. To 15 and 25?
4. To 11 and 20?
5. To 30 and 40?
6. To 36 and 54?
7. To 7 and 9?
8. To 39 and 54?
9. To 6, 8, and 10?
10. To 12, 18, and 30?
11. To 20, 30, and 50?
12. To 42, 56, and 84?
13. To 63, 81, and 99?
14. To 8, 9, and 25?
15. To 7, 49, and 84?

D. 1. A DIVISOR of a number is any entire number which will exactly divide it.

NOTE. Every divisor of a number is a factor of it, and every factor of a number is a divisor of it.

2. A COMMON DIVISOR of two or more numbers is any number which is a divisor of each of them.

ILLUSTRATION. 4 is a common divisor of 4, 8, 12, and 32.

3. The GREATEST COMMON DIVISOR of two or more numbers is the largest number which is a divisor of each of them.

4. Divisors have the following properties:—

1st. A Divisor of a number can contain only such prime factors as are found in that number.

2d. A Common Divisor of two or more numbers can contain only such prime factors as are common to all those numbers.

3d. The Greatest Common Divisor of two or more numbers is the product of *all* the prime factors common to those numbers.

E. 1. What is the greatest common divisor of 24, 36, and 60?

SOLUTION. The greatest common divisor of 24, 36, and 60, is the product of all the prime factors *common* to these numbers.

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3;$$
$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2;$$
$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5;$$

From this it appears that the only common prime factors are 2, 2, and 3. Hence, $2 \times 2 \times 3$, or 12, is the greatest common divisor required.

What is the greatest common divisor—

2. Of 30 and 42?
3. Of 4 and 12?
4. Of 8 and 20?
5. Of 35 and 49?
6. Of 63 and 72?
7. Of 42 and 63?
8. Of 72 and 96?
9. Of 4, 6, and 12?
10. Of 3, 9, and 15?
11. Of 18, 27, and 45?
12. Of 14, 28, and 56?
13. Of 30, 48, and 54?
14. Of 24, 60, and 84?
15. Of 45, 75, and 90?

Find also the greatest common divisor of the numbers in each question under C.

F. 1. A MULTIPLE of a number is any number which contains it as a factor.

NOTE. Every number is a Multiple of all its factors and divisors, and a factor and divisor of all its multiples.

2. A COMMON MULTIPLE of two or more numbers is any number which is a multiple of each of them.

3. The LEAST COMMON MULTIPLE of two or more numbers is the smallest number which is a multiple of each of them.

4. Multiples have the following properties:—

1st. Every Multiple of a number must contain all the prime factors of that number.

2d. Every Common Multiple of two or more numbers must contain all the prime factors of each of those numbers.

3d. The Least Common Multiple of two or more numbers is the smallest number which contains all the prime factors of each of those numbers.

G. What prime factors must every multiple of 18 contain?

SOLUTION. Since $18 = 2 \times 3^2$, every multiple of 18 must contain the factors 2 and 3^2, or 2, 3, and 3.

What prime factors must be contained in every multiple—

2. Of 12?	5. Of 33?	8. Of 48?
3. Of 9?	6. Of 28?	9. Of 60?
4. Of 21?	7. Of 75?	10. Of 86?

H. 1. What is the least common multiple of 36 and 48?

SOLUTION. The least common multiple of 36 and 48 is the smallest number which contains all the prime factors of each of them.

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2.$$
$$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3.$$

We must have the prime factors of 48, which is the same thing as 48 itself. We must also have the factors of 36, which are 2^2 and 3^2, but as we have already taken 2^2 and 3, we have only to introduce the remaining factor 3, which gives 48×3, or $2^4 \times 3^2 = 144$, as the least common multiple required.

2. What is the least common multiple of 9, 24, and 30?

SOLUTION.—The least common multiple of these numbers is the least number which contains all the prime factors of each of them.

$$9 = 3 \times 3.$$
$$24 = 2 \times 2 \times 2 \times 3.$$
$$30 = 2 \times 3 \times 5.$$

We must have the prime factors of 30, or which is the same thing, 30 itself. We must also have the prime factors of 24, which are 2, 2, 2, and 3; but as we have already taken 2 and 3, we have only to introduce the remaining factors 2 and 2, which will give $30 \times 2 \times 2$. We must also have the factors of 9, which are 3 and 3; but as we have already taken one 3, we have only to introduce another 3, which gives $30 \times 2 \times 2 \times 3 = 360$, as the least common multiple required.

What is the least common multiple—

3. Of 8 and 12?
4. Of 6 and 9?
5. Of 4 and 12?
6. Of 7 and 8?
7. Of 12 and 15?
8. Of 16 and 20?
9. Of 2, 4, and 6?
10. Of 9, 12, and 18?
11. Of 10, 25, and 30?
12. Of 4, 6, and 12?
13. Of 3, 4, 5, and 6?
14. Of 8, 10, 12, and 20?

SECTION XVII.

A. 1. Such parts as are obtained by dividing any unit whatever into equal parts are called FRACTIONAL PARTS, and the numbers expressing them are called FRACTIONS.

2. HALVES are such fractional parts as are obtained by dividing a unit into two equal parts.

3. THIRDS are such fractional parts as are obtained by dividing a unit into three equal parts.

What are—

4. Fifths?
5. Sevenths?
6. Ninths?
7. Twelfths?
8. Sixths?
9. Forty-firsts?
10. Fourths?
11. Twentieths?
12. Tenths?

13. Now answer the above questions after the following—

MODEL. Halves are equal parts of such kind that two of them will equal a unit.

B. 1. Fractional parts are generally expressed by drawing a line over the number which shows how many of them must be taken to equal a unit.

ILLUSTRATION. Fourths are expressed by drawing a line above the figure 4, thus, $\overline{4}$. In like manner $\overline{8}$ means eighths, $\overline{21}$ means *twenty-firsts, &c.*

2. The number which thus shows the kind or denomination of the fractional parts is called a DENOMINATOR.

3. In order to show how many fractional parts are taken or considered, a figure called the NUMERATOR is written above the denominator.

ILLUSTRATION. To express five sixths in figures, we write the numerator 5 above the denominator 6, thus, $\frac{5}{6}$.

4. A whole number and a fraction taken together form a MIXED NUMBER.

5. A Mixed Number is written by first writing the whole number, and then writing the fraction.

ILLUSTRATION. The mixed number seven and three eighths is written thus, $7\frac{3}{8}$.

6 How will you write each of the following?

1. Two thirds.
2. Eight ninths.
3. Thirteen nineteenths.
4. Six twenty-firsts.
5. Four and seven tenths.
6. Ten and four fifths.
7. Twelve and eleven twelfths.
8. Six and two thirds.

D. **1.** What does the fraction $\frac{3}{4}$ express?

ANSWER. The fraction three fourths expresses the value of 3 such parts as are obtained by dividing a unit into 4 equal parts.

What does each of the following fractions express?

2. $\frac{2}{3}$. **4.** $\frac{4}{9}$. **6.** $\frac{6}{7}$. **8.** $\frac{16}{21}$.
3. $\frac{5}{6}$. **5.** $\frac{21}{32}$. **7.** $\frac{13}{19}$. **9.** $\frac{23}{24}$.

Explain each of the above fractions after the following—

MODEL. The fraction three fourths expresses the value of three equal parts of such kind that four of them would equal a unit.

SECTION XVIII.

A. Fractions may arise from division, as in the following examples:—

29 = how many times 6?

SOLUTION. 29 = 4 times 6, with 5 remaining, or it equals $4\frac{5}{6}$ times 6.

NOTE. The first part of the above solution should be omitted as soon as the pupil is prepared to give the final answer without it. The entire dividend is here divided, and the *fraction five sixths* is a part of the quotient, and not, like the *remainder* 5, a part of the dividend. Hence it is wrong to say "29 = 4 times 6, with $\frac{5}{6}$ remaining." These distinctions are important, and should be observed in the solutions. See Section XIII., Note under A.

Perform by the above solution the examples under Letters A, B, and C, Section XIII.

B. 1. 41 days = how many weeks?

SOLUTION. Since 7 days = 1 week, 41 days must equal as many weeks as there are times 7 in 41, which are $5\frac{6}{7}$ times. Hence, 41 days = $5\frac{6}{7}$ weeks.

2. Perform by the last solution, the questions under D, Section XIII.

C. 1. How many quarts of vinegar, at 6 cents per quart, can be bought for 53 cents?

SOLUTION. Since 1 quart of vinegar can be bought for 6 cents, as many quarts can be bought for 53 cents as there are times 6 in 53, which are $8\frac{5}{6}$ times. Hence, $8\frac{5}{6}$ quarts *of vinegar, at 6 cents* per quart, can be bought for 53 cents.

2. How many pounds of sugar, at 8 cents per pound, can be bought for 68 cents?

3. How many yards of cloth, at $4 per yard, can be bought for $31?

4. How many bags, each containing 3 bushels, can be filled from 29 bushels of grain?

5. How many hours will it take a horse to trot 33 miles, if he trots 7 miles per hour?

6. How many weeks will it take a man to earn $78, if he earn $9 per week?

7. How many hours will it take a ship to sail 63 miles, if she sail 8 miles per hour?

8. How many months will it take a man, who earns $12 per month, to earn $105?

9. A man put 9 bu. 3 pk. of grain into bags, each holding 1 bu. 3 pk. How many bags could he fill?

10. If a man can earn enough in one day to buy 1 gal. 2 qts. of oil, how many days will it take him to earn enough to buy 13 gal. 1 qt.?

11. Walter gave 25 cents for nuts at 7 cents per quart, and Richard gave 67 cents less than 5 times as many as Walter for nuts at 6 cents per quart. How many quarts did each boy buy?

12. 6 times $9, plus 4 times $7, will buy how many barrels of flour at $9 per barrel?

13. $23 less than 9 times $12 will pay for how many barrels of flour at 7 per barrel?

14. $13 more than the difference between 8 times $7 and 9 times $12, will pay for how many tons of coal at $6 per ton?

15. 6 times $9 plus 8 times $4, diminished by 7 times $3, is $12 less than I paid for cloth at $5 per yard. How many yards did I buy?

SECTION XIX.

A. Fractions may be added, subtracted, multiplied, and divided as whole numbers are. Thus:—

$\frac{2}{7} + \frac{4}{7} = \frac{6}{7}$, just as 2 days + 4 days = 6 days.
$\frac{5}{8} - \frac{3}{8} = \frac{2}{8}$, just as 5 qt. — 3 qt. = 2 qt.
9 times $\frac{3}{4} = \frac{27}{4}$, just as 9 times 3 pecks = 27 pecks.
$\frac{6}{25}$ are contained 4 times in $\frac{24}{25}$, just as 6 lb. are contained 4 times in 24 lb.

B. 1. $7 = *$ fourths?

SOLUTION. Since $1 = 4$ fourths, 7 must equal 7 times 4 fourths, which are 28 fourths. Therefore, $7 = \frac{28}{4}$.

2. $8\frac{4}{5} = *$ fifths?

SOLUTION. Since $1 = 5$ fifths, 8 must equal 8 times 5 fifths, which are 40 fifths, and 4 fifths added are 44 fifths. Hence $8\frac{4}{5} = \frac{44}{5}$.

NOTE. Compare these solutions with those of Section XI., F.

3. $9 = *$ tenths?
4. $5 = *$ thirds?
5. $8 = *$ nineteenths?
6. $4\frac{3}{4} = *$ fourths?
7. $4\frac{3}{7} = *$ seventeenths?
8. $2\frac{8}{19} = *$ nineteenths?
9. $7\frac{13}{17} = *$ seventeenths?
10. $6\frac{9}{11} = *$ elevenths?

C. 1. $\frac{58}{9} = *$ ones?

SOLUTION. Since 9 ninths = 1, 58 ninths must equal as many ones as there are times 9 in 58, which are $6\frac{4}{9}$ times. Hence, $\frac{58}{9} = 6\frac{4}{9}$.

NOTE. Compare this solution with those of Section XIII., D.

2. $\frac{64}{8} = *$ ones?
3. $\frac{27}{3} = *$ ones?
4. $\frac{91}{7} = *$ ones?
5. $\frac{33}{4} = *$ ones?
6. $\frac{67}{9} = *$ ones?
7. $\frac{46}{7} = *$ ones?
8. $\frac{87}{11} = *$ ones?
9. $\frac{104}{12} = *$ ones?
10. $\frac{86}{7} = *$ ones?
11. $\frac{94}{12} = *$ ones?

D. 1. What is the sum of $\frac{9}{11} + \frac{7}{11}$?

FIRST SOLUTION. $\frac{9}{11} + \frac{7}{11} = \frac{16}{11}$, which, since $\frac{11}{11} = 1$, must equal as many ones as there are times 11 in 16, which are $1\frac{5}{11}$ times. Hence, $\frac{9}{11} + \frac{7}{11} = 1\frac{5}{11}$.

SECOND SOLUTION. By uniting two of the $\frac{7}{11}$ with the $\frac{9}{11}$ we have $\frac{9}{11} + \frac{7}{11} = \frac{11}{11} + \frac{5}{11} = 1\frac{5}{11}$.

NOTE. The second form of solution is often more convenient than the first. The pupil should learn both.

2. $\frac{7}{9} + \frac{4}{9}$?
3. $\frac{9}{13} + \frac{11}{13}$?
4. $\frac{7}{12} + \frac{11}{12}$?
5. $\frac{3}{7} + \frac{5}{7} + \frac{4}{7} + \frac{3}{7}$?
6. $\frac{7}{8} + \frac{8}{8} + \frac{4}{8} + \frac{6}{8}$?
7. $\frac{8}{12} + \frac{9}{12} + \frac{7}{12} + \frac{11}{12}$?

1. What is the sum of $6\frac{4}{5} + 8\frac{3}{5}$?

FIRST SOLUTION. $6\frac{4}{5}$ plus 8 are $14\frac{4}{5}$, plus $\frac{3}{5}$ are $14\frac{7}{5} = 15\frac{2}{5}$.

SECOND SOLUTION. $6\frac{4}{5}$ plus 8 are $14\frac{4}{5}$, plus $\frac{3}{5}$ are, by uniting one of the $\frac{3}{5}$ with the $\frac{4}{5}$, $14\frac{5}{5} + \frac{2}{5} = 15\frac{2}{5}$.

NOTE. Some prefer to add the whole numbers, and then the fractions, thus, $6 + 8 = 14$; $\frac{4}{5} + \frac{3}{5} = 1\frac{2}{5}$; $14 + 1\frac{2}{5} = 15\frac{2}{5}$. Hence, $6\frac{4}{5} + 8\frac{3}{5} = 15\frac{2}{5}$.

2. $9\frac{4}{7} + 6\frac{5}{7}$?
3. $3\frac{8}{9} + 4\frac{7}{9}$?
4. $4\frac{8}{11} + 2\frac{6}{11}$?
5. $5\frac{17}{20} + 8\frac{18}{20}$?
6. $8\frac{9}{10} + 7\frac{5}{10} + 6\frac{9}{10}$?
7. $5\frac{2}{3} + 7\frac{2}{3} + 4\frac{1}{3} + 5\frac{2}{3}$?
8. $8\frac{4}{5} + 2\frac{3}{5} + 7\frac{4}{5} + 3\frac{3}{5}$?
9. $5\frac{7}{8} + 6\frac{4}{8} + 5\frac{7}{8} + 4\frac{6}{8}$?

E. 1. How many are $6\frac{7}{8} - 3\frac{5}{8}$?

SOLUTION. $6\frac{7}{8} - 3 = 3\frac{7}{8}$; $3\frac{7}{8} - \frac{5}{8} = 3\frac{2}{8}$. Hence, $6\frac{7}{8} - 3\frac{5}{8} = 3\frac{2}{8}$.

How many are—

2. $8\frac{6}{7} - 2\frac{1}{7}$?
3. $14\frac{8}{9} - 7\frac{4}{9}$?
4. $18\frac{7}{12} - 4\frac{5}{12}$?
5. $16\frac{4}{5} - 9\frac{3}{5}$?
6. $15\frac{13}{21} - 4\frac{2}{21}$?
7. $38\frac{43}{75} - 29\frac{37}{75}$?

F. 1. What is the value of $1 - \frac{7}{12}$?

SOLUTION. $1 = \frac{12}{12}$, and $\frac{12}{12} - \frac{7}{12} = \frac{5}{12}$.

2. What is the value of $8 - \frac{9}{20}$?

SOLUTION. Since $8 = 7\frac{20}{20}$, $8 - \frac{9}{20}$ must equal $7\frac{20}{20} - \frac{9}{20} = 7\frac{11}{20}$.

3. $1 - \frac{3}{5}$?
4. $1 - \frac{13}{25}$?
5. $1 - \frac{17}{28}$?
6. $4 - \frac{9}{10}$?
7. $56 - \frac{3}{28}$?
8. $24 - \frac{15}{49}$?
9. $3 - \frac{5}{7}$?
10. $8 - \frac{2}{11}$?
11. $9 - \frac{5}{8}$?

G. 1. How many are $8\frac{7}{20} - \frac{13}{20}$?

FIRST SOLUTION. Since $8\frac{7}{20} = 7 + 1\frac{7}{20} = 7\frac{27}{20}$, it follows that $8\frac{7}{20} - \frac{13}{20} = 7\frac{27}{20} - \frac{13}{20} = 7\frac{14}{20}$.

SECOND SOLUTION. $8\frac{7}{20} - \frac{13}{20} = 8\frac{7}{20} - \frac{7}{20} - \frac{6}{20} = 8 - \frac{6}{20} = 7\frac{14}{20}$.

What is the value of—

2. $4\frac{1}{3} - \frac{2}{3}$?
3. $16\frac{3}{8} - \frac{7}{8}$?
4. $14\frac{5}{9} - \frac{8}{9}$?
5. $21\frac{4}{11} - \frac{6}{11}$?
6. $48\frac{1}{13} - \frac{12}{13}$?
7. $14\frac{3}{7} - \frac{5}{7}$?
8. $9\frac{8}{25} - \frac{13}{25}$?
9. $4\frac{17}{40} - \frac{33}{40}$?
10. $38\frac{21}{75} - \frac{59}{75}$?

H. 1. How many are $23\frac{5}{13} - 16\frac{9}{13}$?

FIRST SOLUTION. $23\frac{5}{13} - 16 = 7\frac{5}{13} = 6\frac{18}{13}$; $6\frac{18}{13} - \frac{9}{13} = 6\frac{9}{13}$.

SECOND SOLUTION. $23\frac{5}{13} - 16 = 7\frac{5}{13}$; $7\frac{5}{13} - \frac{9}{13} = 7\frac{5}{13} - \frac{5}{13} - \frac{4}{13} = 7 - \frac{4}{13} = 6\frac{9}{13}$.

How many are—

2. $9\frac{1}{6} - 7\frac{5}{6}$?
3. $4\frac{3}{10} - \frac{8}{10}$?
4. $16\frac{4}{12} - 5\frac{7}{12}$?
5. $43\frac{2}{9} - 17\frac{6}{9}$?
6. $23\frac{4}{11} - 13\frac{9}{11}$?
7. $8\frac{2}{17} - 3\frac{14}{17}$?
8. $64\frac{2}{3} + 4\frac{2}{3} - 8\frac{2}{3}$?
9. $23\frac{3}{4} + 17\frac{2}{4} - 8\frac{3}{4}$?

I. 1. George gave William $\frac{3}{8}$ of an apple, Joseph $\frac{2}{8}$ of it, and ate the rest himself. What part of it did he eat?

2. Edward earned $\frac{3}{4}$ of a dollar by picking blackberries, $\frac{2}{4}$ of a dollar by picking strawberries, and $\frac{3}{4}$ of a dollar by picking blueberries. How much did he earn in all?

3. Who can tell whether the sum of $8\frac{2}{5} + 5\frac{4}{5} + 3\frac{3}{5} + 7\frac{3}{5}$ is greater or less than $24\frac{4}{5}$, and how much?

4. $5\frac{8}{9}$ acres were sold from a lot containing $8\frac{4}{9}$ acres. How many acres were left?

5. Mr. Stone gave $\frac{5}{12}$ of his money for a lot of land, and $\frac{5}{12}$ for a horse. What part of it had he left?

6. Isaac caught three nice trout. The first weighed $3\frac{7}{16}$ lb., the second weighed $2\frac{14}{16}$ lb., and the third weighed $1\frac{15}{16}$ lb. How much did they all weigh?

7. Mr. Davis owns $\frac{7}{20}$ of a vessel, Mr. Mason owns $\frac{6}{20}$, and Mr. Allen owns the rest. What part of the vessel does Mr. Allen own?

8. Julia's father gave her $\frac{28}{50}$ of a dollar, her mother gave her $\frac{12}{50}$, her brother gave her $\frac{12}{50}$, and her uncle gave her enough to make up 2 dollars. How much did her uncle give her?

9. Farmer Brown had $17\frac{4}{23}$ tons of hay in his barn, and $15\frac{7}{23}$ tons in his stacks. How many tons had he in both? He moved $3\frac{15}{23}$ tons from his stacks into his barn. How

many tons were then in his barn? How many in his stacks?

10. A man paid $11\frac{1}{8}$ dollars for a coat, $3\frac{7}{8}$ dollars for a pair of pants, and $2\frac{5}{8}$ dollars for a vest, giving in payment a twenty-dollar bill. How much ought he to receive back?

SECTION XX.

1. How many are 9 times $\frac{6}{7}$?

SOLUTION. 9 times $\frac{6}{7} = \frac{54}{7}$, which, since 7 sevenths equal 1, must equal as many ones as there are times 7 in 54, which are $7\frac{5}{7}$ times. Hence, 9 times $\frac{6}{7} = 7\frac{5}{7}$.

ABBREVIATED SOLUTION. 9 times $\frac{6}{7} = \frac{54}{7} = 7\frac{5}{7}$.

How many are—

2. $4 \times \frac{6}{12}$?
3. $8 \times \frac{4}{5}$?
4. $4 \times \frac{8}{9}$?
5. $7 \times \frac{11}{12}$?
6. $9 \times \frac{7}{8}$?
7. $4 \times \frac{9}{12}$?
8. $5 \times \frac{8}{9}$?
9. $6 \times \frac{11}{12}$?
10. $12 \times \frac{9}{11}$?
11. $14 \times \frac{6}{7}$?
12. $15 \times \frac{8}{13}$?
13. $11 \times \frac{7}{12}$?

How many are $6 \times 7\frac{3}{5}$?

SOLUTION. 6 times $7\frac{3}{5}$ = 6 times 7 + 6 times $\frac{3}{5}$; 6 times 7 = 42, and 6 times $\frac{3}{5} = \frac{18}{5} = 3\frac{3}{5}$, which, added to 42, gives $45\frac{3}{5}$. Hence, 6 times $7\frac{3}{5} = 45\frac{3}{5}$.

How many are—

14. $9 \times 8\frac{3}{5}$?
15. $8 \times 9\frac{3}{4}$?
16. $7 \times 4\frac{8}{9}$?
17. $4 \times 6\frac{12}{13}$?
18. $6 \times 8\frac{9}{10}$?
19. $5 \times 9\frac{5}{7}$?
20. $4 \times 8\frac{11}{12}$?
21. $6 \times 14\frac{9}{10}$?
22. $12 \times 21\frac{3}{5}$?
23. $9 \times 63\frac{4}{9}$?
24. $9 \times 59\frac{5}{12}$?
25. $11 \times 32\frac{7}{11}$?

B. 1. $12\frac{3}{4} = *$ times $2\frac{1}{4}$?

SOLUTION. $12\frac{3}{4} = \frac{51}{4}$; $2\frac{1}{4} = \frac{9}{4}$; and $\frac{51}{4}$ contains $\frac{9}{4}$ as many times as 51 contains 9, which are $5\frac{6}{9}$ times. Hence, $12\frac{3}{4} = 5\frac{6}{9}$ times $2\frac{1}{4}$.

2. What is the quotient of $8\frac{2}{3} \div 3$?

SOLUTION. $8\frac{2}{3} = \frac{26}{3}$; $3 = \frac{9}{3}$; and the quotient of $\frac{26}{3} \div \frac{9}{3}$ = the quotient of $26 \div 9 = 2\frac{8}{9}$. Hence, the quotient of $8\frac{2}{3} \div 3 = 2\frac{8}{9}$.

3. $7\frac{1}{2} = *$ times $1\frac{1}{2}$?
4. $4\frac{2}{3} = *$ times $1\frac{2}{3}$?
5. $8\frac{4}{7} = *$ times $1\frac{2}{7}$?
6. $9\frac{3}{5} = *$ times $2\frac{2}{5}$?
7. $5\frac{6}{9} = *$ times $\frac{7}{9}$?
8. $7\frac{1}{5} \div 1\frac{4}{5}$?
9. $8\frac{3}{4} \div 2\frac{1}{4}$?
10. $9\frac{1}{2} \div 3\frac{1}{2}$?
11. $6\frac{3}{7} \div 1\frac{2}{7}$?
12. $8\frac{3}{4} \div 1\frac{3}{4}$?

C. 1. How much will 7 yards of cloth cost at $9\frac{4}{7}$ dollars per yard?

2. A man gave 6 cords of wood at $\$4\frac{2}{3}$ per cord for raisins at \$7 per cask. How many casks did he buy?

3. How many yards of cloth at \$3 per yard can be bought for 8 bbls. of cider at $\$3\frac{3}{4}$ per barrel?

4. I gave 9 firkins of butter at $\$4\frac{4}{8}$ per firkin for flour at \$7 per barrel. How many barrels did I buy?

5. How many shade trees, worth $\frac{1}{4}$ of a dollar apiece, can be bought for $5\frac{1}{4}$ dollars?

6. How many skeins of silk worth $\frac{1}{8}$ of a dime per skein can be bought for $6\frac{2}{5}$ dimes?

7. How many baskets each containing $\frac{2}{7}$ of a bushel can be filled from $8\frac{3}{7}$ bushels of peaches?

8. I paid $6\frac{2}{3}$ dollars for grain at $\frac{1}{3}$ of a dollar per bushel. How many bushels did I buy? I put the grain into bags each holding $1\frac{1}{4}$ bushels. How many bags did I fill?

9. A farmer exchanged 5 barrels of apples at $1\frac{6}{7}$ dollars per barrel, for oil at $1\frac{1}{7}$ dollars per gallon. How many gallons of oil did he get?

10. How many pounds of tea worth $\frac{5}{8}$ of a dollar per pound, should be given for three books worth $2\frac{3}{8}$ dollars apiece?

11. A man who had $12\frac{2}{3}$ bushels of potatoes, used $2\frac{2}{3}$ bushels, and then sold the rest at $\frac{7}{9}$ of a dollar per bushel, receiving in payment cloth at $\frac{8}{9}$ of a dollar per yard. How many yards of cloth did he receive?

12. I bought 8 barrels of flour at $\$7\frac{1}{9}$ per barrel, and gave in payment 12 cords of wood at $\$4\frac{1}{9}$ per cord, and the rest in apples at $\frac{4}{9}$ of a dollar per bushel. How many bushels of apples did I give?

13. Rufus has $\$3\frac{5}{8}$, and Thomas has $\$4\frac{1}{8}$ less than 7 times as much as Rufus. How many dollars has Thomas?

14. Franklin expended $\$8\frac{1}{3}$ for corn at $\frac{1}{3}$ of a dollar per bushel. How many bushels did he buy, and how many days would it last him if he should use $1\frac{1}{3}$ bushels per day?

SECTION XXI.

A. 1. What fractional part of 5 is 1?

ANSWER. 1 is $\frac{1}{5}$ of 5, because 5 times $1 = 5$.

What fractional part of—

2. 7 is 1?	**4.** 9 is 1?	**6.** 8 is 1?
3. 2 is 1?	**5.** 10 is 1?	**7.** 3 is 1?

B. 1. *What* fractional part of 8 is 3?

SOLUTION. Since $1 = \frac{1}{8}$ of 8, 3 must equal $\frac{3}{8}$ of 8.

What fractional part of—

2. 12 is 7?	**5.** 10 is 9?	**8.** 13 is 11?
3. 9 is 4?	**6.** 9 is 10?	**9.** 8 is 5?
4. 7 is 2?	**7.** 12 is 5?	**10.** 5 is 8?

C. **1.** What fractional part of 9 qt. is 4 qt.?

SOLUTION. 4 qt. is the same fractional part of 9 qt. that 4 is of 9, which is $\frac{4}{9}$. Hence, 4 qt. $= \frac{4}{9}$ of 9 qt.

What fractional part of—

2. 11 lb. is 7 lb?	**4.** \$5 is \$3?
3. 7 lb. is 11 lb?	**5.** £12 is £5?

1. What fractional part of $\frac{5}{7}$ is $\frac{2}{7}$?

SOLUTION. $\frac{2}{7}$ is the same fractional part of $\frac{5}{7}$ that 2 is of 5, which is $\frac{2}{5}$. Hence, $\frac{2}{7} = \frac{2}{5}$ of $\frac{5}{7}$.

What fractional part of—

2. $\frac{4}{9}$ is $\frac{3}{9}$?	**4.** $\frac{4}{21}$ is $\frac{9}{21}$?	**6.** $\frac{8}{11}$ is $\frac{3}{11}$?
3. $\frac{6}{11}$ is $\frac{5}{11}$?	**5.** $\frac{5}{8}$ is $\frac{7}{8}$?	**7.** $\frac{9}{12}$ is $\frac{7}{12}$?

D. **1.** What fractional part of $2\frac{2}{3}$ is $10\frac{1}{3}$?

SOLUTION. $2\frac{2}{3} = \frac{8}{3}$; $10\frac{1}{3} = \frac{31}{3}$; $\frac{31}{3}$ is the same fractional part of $\frac{8}{3}$ that 31 is of 8, which is $\frac{31}{8}$ or $3\frac{7}{8}$. Hence, $10\frac{1}{3} = \frac{31}{8}$ of $2\frac{2}{3}$, or it equals $3\frac{7}{8}$ times $2\frac{2}{3}$.

What fractional part of—

2. $4\frac{1}{2}$ is $2\frac{1}{2}$?	**7.** 3 da. is 1 wk?
3. $7\frac{1}{7}$ is $2\frac{2}{7}$?	**8.** 4 sq. ft. is 1 sq. yd.?
4. $9\frac{3}{8}$ is $2\frac{5}{8}$?	**9.** 1 pk. 1 qt. is 3 pk. 7 qt.?
5. $\frac{4}{5}$ is 1?	**10.** 3 yd. 1 ft is 8 yd. 2 ft.?
6. $\frac{2}{3}$ is 1?	**11.** 9 d. 1 qr. is 3 d. 2 qr.?

E. What fractional part—

1. Of the cost of 7 yd. is the cost of 4 yd.?
2. Of the cost of 3 acres is the cost of 4 acres?
3. Of the cost of $\frac{3}{7}$ is the cost of $\frac{2}{7}$?
4. Of the cost of $\frac{8}{9}$ is the cost of $\frac{4}{9}$?
5. Of the cost of 1 bushel is the cost of 3 pecks?
6. Of the cost of dwt. is the cost of 13 gr.?
7. Of the cost of 1 is the cost of $\frac{3}{4}$?
8. Of the cost of 1 is the cost of $\frac{5}{7}$?
9. Of the cost of 3 qt. is the cost of 1 pk.?
10. Of the cost of 7 $\frac{7}{3}$ is the cost of 1 ℔?
11. Of the cost of $\frac{3}{5}$ is the cost of 1?
12. Of the cost of $\frac{8}{17}$ is the cost of 1?

13. George and John together bought 8 apples, of which George had 5. What part of the cost ought each to pay?

14. Reuben, Walter, and Joel together bought a lot of 25 oranges, of which Reuben took 7, Walter took 6, and Joel the remainder. What part of the cost ought each to pay?

SECTION XXII.

A. 1. What is $\frac{1}{3}$ of 63?

ANSWER. $\frac{1}{3}$ of 63 is 7, because 9 times 7 = 63, or because 7 taken 9 times = 63.

2. $\frac{1}{3}$ of 24?
3. $\frac{1}{5}$ of 25?
4. $\frac{1}{7}$ of 49?
5. $\frac{1}{8}$ of 42?
6. $\frac{1}{8}$ of 64?
7. $\frac{1}{9}$ of 81?

8. What is $\frac{3}{8}$ of 72?

FIRST SOLUTION. $\frac{3}{8}$ of 72 = 3 times $\frac{1}{8}$ of 72; $\frac{1}{8}$ of 72 = *9, and 3 times* 9 = 27. Hence, $\frac{3}{8}$ of 72 = 27.

SECOND SOLUTION. $\frac{1}{8}$ of 72 = 9, and $\frac{3}{8}$ of 72 must equal 3 times 9, which are 27. Hence, $\frac{3}{8}$ of 72 = 27.

What is—

9. $\frac{4}{5}$ of 40?	12. $\frac{7}{8}$ of 56?	15. $\frac{5}{9}$ of 36?
10. $\frac{2}{3}$ of 18?	13. $\frac{9}{10}$ of 80?	16. $\frac{8}{9}$ of 63?
11. $\frac{5}{8}$ of 48?	14. $\frac{7}{9}$ of 72?	17. $\frac{5}{6}$ of 54?

B. 1. How many are $4\frac{2}{3}$ times 9?

SOLUTION. $4\frac{2}{3}$ times 9 = 4 times 9 + $\frac{2}{3}$ of 9 = 4 times 9 = 36; $\frac{2}{3}$ of 9 = 2 times $\frac{1}{3}$ of 9; $\frac{1}{3}$ of 9 = 3, and 2 times 3 = 6, which, added to 36 = 42. Hence, $4\frac{2}{3}$ times 9 = 42.

ABBREVIATED SOLUTION. 4 times 9 = 36. $\frac{1}{3}$ of 9 = 3, and $\frac{2}{3}$ of 9 must equal 2 times 3, or 6, which, added to 36 = 42. Hence, $4\frac{2}{3}$ times 9 = 42.

How many are—

2. $7\frac{1}{2}$ times 6?	6. $\frac{5}{6}$ of 54 = * times 3?
3. $9\frac{3}{4}$ times 8?	7. $\frac{6}{7}$ of 42 = * times 9?
4. $8\frac{4}{5}$ times 10?	8. $\frac{3}{5}$ of 40 = * times 7?
5. $6\frac{2}{3}$ times 9?	9. $8\frac{2}{3}$ times 9 = * times 10?

10. $\frac{4}{5}$ of 30 + $\frac{2}{7}$ of 56 = * times 5?
11. $\frac{5}{9}$ of 45 + $\frac{4}{5}$ of 25 = * times 9?
12. $\frac{4}{7}$ of 49 + $\frac{2}{9}$ of 36 = * times 6?
13. $\frac{3}{8}$ of 72 + $\frac{3}{9}$ of 63 = * times 9?

C. 1. If 8 pictures cost 72 cents, how much will 5 cost?

SOLUTION. If 8 pictures cost 72 cents, 1 picture will cost $\frac{1}{8}$ of 72 cents, which is 9 cents, and 5 pictures will cost 5 times 9 cents, which are 45 cents. Hence, 5 pictures will cost 45 cents, if 8 cost 72 cents.

How much will—

2. 3 sheep cost if 7 cost 49 dollars?
3. 8 pounds of meat cost if 7 pounds cost 84 cents?
4. 12 milk-cans hold if 3 hold 24 quarts?
5. 9 stacks contain if 12 contain 144 tons?
6. 3 quarts cost at 32 cents per gallon?

PARTIAL SOLUTION. If 1 gallon, or 4 quarts, costs 32 cents, 1 quart or $\frac{1}{4}$ of a gallon, will cost $\frac{1}{4}$ of 32 cents, which is, &c.

How much will—

7. 7 qt. of meal cost at 24 cents per peck?
8. 3 gills of oil cost at 40 cents per quart?
9. 11 oz. of tea cost at 64 cents per pound?
10. 1 A. 3 R. (or 7 R.) of land cost at $112 per acre?
11. 3 yd. 2 ft. of cloth cost at 36 cents per yard?
12. 1 gal. of molasses cost if 1 gal. 2 qt. cost 42 cents?
13. 1 pk. 4 qt. of nuts cost if 1 pk. 3 qt. cost 88 cents?
14. 3 gal. 2 qt. 1 pt. of vinegar cost if 2 gal. 2 qt. cost 40 cents?
15. 2 sq. yd. 3 sq. ft. of land cost if 3 sq. yd. 3 sq. ft. cost $1.20?
16. $\frac{5}{8}$ of a yd. of linen cost at 54 cents per yard?
17. $\frac{5}{9}$ of an acre of land cost at $108 per acre?
18. $5\frac{3}{4}$ bbl. of flour cost at $8 per barrel?
19. $3\frac{5}{8}$ yards of muslin cost at 32 cents per yard?
20. A lot 9 rods long and $8\frac{2}{3}$ rods wide contain?
21. A blackboard 12 ft. long and $2\frac{5}{8}$ ft wide contain?
22. $5\frac{2}{3}$ tons of hay cost if 8 tons cost $72?
23. $4\frac{2}{3}$ lumps of butter weigh if 9 lumps weigh 81 pounds?
24. *$5\frac{1}{4}$ months'* labor be worth at $28 per month?

25. How many pounds of sugar at 8 cents per pound can be bought for $2\frac{3}{4}$ dozen of eggs at 24 cents per dozen?

26. If John can earn 63 cents in 7 hours, how many hours will it take him to earn enough to buy $6\frac{4}{5}$ quarts of filberts at 80 cents per peck?

27. Mr. Wingate owns $7\frac{2}{3}$ acres of land, Mr. Paine owns 6 acres less than 6 times as much as Mr. Wingate, and Mr. Jackson owns $5\frac{1}{4}$ acres more than $\frac{1}{5}$ as much as Mr. Paine owns. How many acres does each own? How many do they all own?

28. I have earned $\$6\frac{7}{8}$, Harry has earned $\$20\frac{3}{8}$ less than 5 times as much as I have earned, and Cyrus has earned twice as much as both of us. How much has Cyrus earned?

SECTION XXIII.

A. **1.** What is $\frac{1}{4}$ of 3?

SOLUTION. $\frac{1}{4}$ of 3 is $\frac{3}{4}$ of 1, or $\frac{3}{4}$, for if 3 equal things should be divided into 4 equal parts, one of those parts would equal $\frac{3}{4}$ of one thing.

NOTE.—This may be illustrated to the eye by taking 3 equal lines and dividing them into 4 equal parts, arranged as in the figure at the left. One part will then contain $\frac{1}{4}$ of 3 lines, which, as will be seen, is equivalent to $\frac{3}{4}$ of a line.

2. $\frac{1}{8}$ of 7? **4.** $\frac{1}{5}$ of 3? **6.** $\frac{1}{8}$ of 5?
3. $\frac{1}{9}$ of 3? **5.** $\frac{1}{7}$ of 1? **7.** $\frac{1}{8}$ of 4?

B. From the preceding exercises, it follows that $\frac{2}{5}$, or $\frac{3}{5}$ of $1 = \frac{1}{5}$ of 3; that $\frac{7}{9}$, or $\frac{7}{9}$ of $1 = \frac{1}{9}$ of 7, &c. Hence,

$\frac{3}{5}$ of any number must equal 3 times $\frac{1}{5}$ of that number, or $\frac{1}{5}$ of 3 times that number; $\frac{8}{9}$ of any number must equal 8 times $\frac{1}{9}$ of that number, or $\frac{1}{9}$ of 8 times that number, &c., &c.

1. What is $\frac{7}{8}$ of 5?

FIRST SOLUTION. $\frac{7}{8}$ of 5 = 7 times $\frac{1}{8}$ of 5; $\frac{1}{8}$ of 5 = $\frac{5}{8}$, and 7 times $\frac{5}{8}$ = $\frac{35}{8}$ = $4\frac{3}{8}$. Hence, $\frac{7}{8}$ of 5 = $4\frac{3}{8}$.

SECOND SOLUTION. $\frac{7}{8}$ of 5 = $\frac{1}{8}$ of 7 times 5; 7 times 5 = 35, and $\frac{1}{8}$ of 35 = $4\frac{3}{8}$. Hence, $\frac{7}{8}$ of 5 = $4\frac{3}{8}$

NOTE. The pupil should master the first solution, and then the second, and afterwards be required to use in each example the one best adapted to that example.

What is—

2. $\frac{3}{4}$ of 2? 4. $\frac{2}{9}$ of 8? 6. $\frac{6}{11}$ of 7?
3. $\frac{5}{7}$ of 6? 5. $\frac{11}{12}$ of 8? 7. $\frac{3}{5}$ of 4?

C. 1. What is $\frac{1}{8}$ of 67?

FIRST SOLUTION. $\frac{1}{8}$ of 67 = $\frac{1}{8}$ of 64 + $\frac{1}{8}$ of 3; $\frac{1}{8}$ of 64 = 8; $\frac{1}{8}$ of 3 = $\frac{3}{8}$, which, added to 8 = $8\frac{3}{8}$. Hence, $\frac{1}{8}$ of 67 = $8\frac{3}{8}$.

SECOND SOLUTION. $\frac{1}{8}$ of 67 = $\frac{1}{8}$ of 64 + $\frac{1}{8}$ of 3 = $8\frac{3}{8}$.

THIRD SOLUTION. $\frac{1}{8}$ of 67 = $8\frac{3}{8}$.

NOTE. The first and second solutions are chiefly valuable as a preparation for the third.

What is—

2. $\frac{1}{9}$ of 43? 5. $\frac{2}{3}$ of 17? 8. $\frac{7}{10}$ of 89?
3. $\frac{1}{6}$ of 28? 6. $\frac{4}{7}$ of 20? 9. $\frac{4}{5}$ of 43?
4. $\frac{1}{4}$ of 17? 7. $\frac{5}{9}$ of 80? 10. $\frac{3}{4}$ of 27?

D. 1. $\frac{1}{6}$ of $52\frac{2}{7}$?

SOLUTION. $\frac{1}{6}$ of $52\frac{2}{7} = \frac{1}{6}$ of $48 + \frac{1}{6}$ of $4\frac{2}{7}$; $\frac{1}{6}$ of $48 = 8$; $\frac{1}{6}$ of $4\frac{2}{7}$ or of $\frac{30}{7} = \frac{5}{7}$, which, added to $8 = 8\frac{5}{7}$. Hence, $\frac{1}{6}$ of $52\frac{2}{7} = 8\frac{5}{7}$.

What is—

2. $\frac{1}{5}$ of $17\frac{1}{2}$?
3. $\frac{1}{9}$ of $41\frac{5}{8}$?
4. $\frac{1}{7}$ of $66\frac{8}{9}$?
5. $\frac{3}{4}$ of $26\frac{2}{5}$?
6. $\frac{2}{9}$ of $86\frac{1}{7}$?
7. $\frac{5}{8}$ of 75?
8. $\frac{1}{5}$ of 17 qt. 1 pt.?
9. $\frac{1}{9}$ of 41 pk. 5 qt.?
10. $\frac{1}{7}$ of 66 sq. yd. 8 sq. ft.?
11. $\frac{4}{9}$ of 49 bu. 2 pk.?
12. $\frac{7}{8}$ of 58 yd. 2 ft.?
13. $\frac{2}{3}$ of 26 wk. 4 days?

E. 1. $\frac{8}{9}$ of $45 = *$ times $\frac{1}{8}$ of 42?

SOLUTION. $\frac{8}{9}$ of $45 = 8$ times $\frac{1}{9}$ of 45; $\frac{1}{9}$ of $45 = 5$, and 8 times $5 = 40$; $\frac{1}{8}$ of $42 = 7$, and 7 is contained in 40, $5\frac{5}{7}$ times. Therefore, $\frac{8}{9}$ of $45 = 5\frac{5}{7}$ times $\frac{1}{8}$ of 42.

2. $\frac{7}{10}$ of $80 = *$ times $\frac{1}{8}$ of 64?
3. $\frac{5}{9}$ of $36 = *$ times $\frac{1}{8}$ of 24?
4. $\frac{7}{8}$ of $72 = *$ times $\frac{1}{4}$ of 32?
5. $\frac{2}{5}$ of $40 = *$ times $\frac{5}{6}$ of 12?
6. $\frac{5}{2}$ of $16 = *$ times $\frac{2}{3}$ of 9?
7. $\frac{2}{7}$ of $25 = *$ times $\frac{1}{7}$ of 9?
8. $\frac{3}{8}$ of $19 = *$ times $\frac{1}{8}$ of 7?
9. $\frac{4}{5}$ of $14 = *$ times $\frac{1}{5}$ of 8?

F. How much will—

1. 3 inkstands cost if 7 inkstands cost 45 cents?
2. 5 melons cost if 9 cost 77 cents?
3. 7 weeks' board cost if 8 cost $27?
4. 3 quarts of cranberries cost at 53 cents per peck?
5. 1 qt. 1 pt. of burning fluid cost at 79 cents per gallon?

6. 1 pk. of meal cost if 1 pk. 3 qt. cost 28 cents?
7. $\frac{4}{5}$ of a yard of cloth cost at 27 cents per yard?
8. $\frac{2}{3}$ of a pound of tea cost at 67 cents per pound?
9. $\frac{5}{9}$ of an acre of land cost at $100 per acre?
10. $3\frac{2}{7}$ weeks' board cost at $10 per week?
11. $7\frac{2}{3}$ boxes of strawberries cost at 25 cents per box?
12. 2 bu. 3 pk. of salt cost at 40 cents per bushel?
13. 7 gal. 2 qt. of molasses cost at 30 cents per gallon?
14. 1 lb. 5 oz. of tea cost at 68 cents per pound?
15. $\frac{4}{5}$ of a yard of muslin cost at $27\frac{1}{2}$ cents per yard?
16. $\frac{3}{4}$ of a bushel of apples cost at $67\frac{2}{3}$ cents per bushel?
17. $\frac{3}{4}$ of a bushel of oats cost at $39\frac{5}{9}$ cents per bushel?
18. $\frac{6}{8}$ of an acre of land cost at $93\frac{5}{7}$ per acre?
19. $\frac{5}{9}$ of a furlong = how many rods?

ABBREVIATED SOLUTION. Since 40 rods = 1 furlong, $\frac{5}{9}$ of a furlong must equal $\frac{5}{9}$ of 40 rods, which is $22\frac{2}{9}$ rods.

20. $\frac{5}{9}$ of a qr. = how many pounds?
21. $\frac{6}{7}$ of a cu. yd. = how many cubic feet?
22. $\frac{5}{9}$ of an hour = how many minutes?
23. $\frac{4}{5}$ of a day = how many hours?
24. $\frac{2}{3}$ of a bu. = how many pk., qt., &c.?

ABBREVIATED SOLUTION. Since 1 bu. = 4 pk., $\frac{2}{3}$ of a bu. must equal $\frac{2}{3}$ of 4 pk., which is $2\frac{2}{3}$ pk. But since 1 pk. = 8 qt., $\frac{2}{3}$ of a pk. must equal $\frac{2}{3}$ of 8 qt., which is $5\frac{1}{3}$ qt. Since 1 qt. = 2 pt., $\frac{1}{3}$ of a qt. must equal $\frac{1}{3}$ of 2 pt., which is $\frac{2}{3}$ of a pt. Since 1 pt. = 4 gills, $\frac{2}{3}$ of a pt. must equal $\frac{2}{3}$ of 4 gills, which is $2\frac{2}{3}$ gills. Therefore, $\frac{2}{3}$ of a bu. = 2 pk. 5 qt. 0 pt. $2\frac{2}{3}$ gills.

25. $\frac{5}{7}$ of a £ = how many s., d., and qr.?
26. $\frac{4}{5}$ of a lb. = how many oz. and dwt.?
27. $\frac{6}{7}$ of a ton = how many cwt., qr., lb.?
28. $\frac{5}{8}$ *of a sq.* yd. = how many sq. ft., sq. in.?

29. $\frac{5}{7}$ of a lb. = how many oz., dwt., gr.?

30. $\frac{4}{9}$ of a wk. = how many da., hr., &c.?

31. $\frac{3}{5}$ of a lb. = how many ℥, ʒ, &c.?

32. $\frac{7}{9}$ of a £ = how many s. d., qr.?

33. Herman gathered 8 quarts of walnuts, Amos gathered $3\frac{1}{3}$ quarts more than $4\frac{1}{8}$ times as many as Herman. How many did Amos gather?

34. If 4 pounds of rice are worth 24 cents, how many pounds of rice ought to be given for $7\frac{3}{4}$ pounds of sugar worth 8 cents per pound?

35. How many half-pounds of coffee worth 16 cents per pound would be given for $2\frac{1}{4}$ bushels of oats worth 44 cents per bushel?

36. I gave 4 pk. 3 qt. of nuts worth 24 cents per peck, for eggs at 12 cents per dozen. How many dozen eggs did I receive?

37. I worked $2\frac{5}{6}$ weeks at $10 per week, receiving $\frac{2}{5}$ of my wages in cash, and the rest in cloth at $3 per yard. How many yards of cloth did I receive?

38. I bought $1\frac{7}{9}$ bushels of apples at 72 cents per bushel, and $\frac{3}{4}$ as many pears at $1\frac{1}{3}$ times as many cents per bushel. What was the amount of my purchase?

39. $3\frac{5}{8}$ times the cost of $\frac{4}{7}$ of a peck of apples at $1.12 per bushel is 4 cents more than the price of a bushel of corn. What is the price of $2\frac{2}{3}$ bushels of corn?

40. If corn is worth 75 cents per bushel, and a bushel of oats is worth $20\frac{1}{2}$ cents less than $\frac{5}{8}$ as much as $1\frac{1}{3}$ bushels of corn, how much is $\frac{1}{2}$ of a bushel of oats worth?

41. If Erastus can earn 44 cents per day, and Huron can earn in one day $3\frac{4}{5}$ cents more than $\frac{3}{5}$ as much as Erastus can earn in $1\frac{3}{4}$ days, how many cents can Huron earn in $1\frac{3}{4}$ days?

SECTION XXIV.

A. 1. $3 = \frac{1}{8}$ of what number?

FIRST SOLUTION. $3 = \frac{1}{8}$ of 8 times 3, or of 24.

SECOND SOLUTION. If 3 is $\frac{1}{8}$ of some number, $\frac{8}{8}$ of the number must be 8 times 3, or 24. Hence, $3 = \frac{1}{8}$ of 24.

2. $9 = \frac{1}{7}$ of $*$?
3. $7 = \frac{1}{6}$ of $*$?
4. $3 = \frac{1}{4}$ of $*$?
5. $\frac{5}{6} = \frac{1}{6}$ of $*$?
6. $\frac{2}{3} = \frac{1}{9}$ of $*$?
7. $\frac{8}{9} = \frac{1}{7}$ of $*$?
8. $9\frac{1}{2} = \frac{1}{3}$ of $*$?
9. $7\frac{2}{3} = \frac{1}{5}$ of $*$?
10. 2 bu. 3 pk. $= \frac{1}{3}$ of $*$?
11. 5 wk. 3 da. $= \frac{1}{8}$ of $*$?

B. 1. $17 = \frac{3}{4}$ of what number?

SOLUTION. If $17 = \frac{3}{4}$ of some number, $\frac{1}{4}$ of that number must be $\frac{1}{3}$ of 17, which is, $5\frac{2}{3}$, and $\frac{4}{4}$ of the number must be 4 times $5\frac{2}{3}$, which is, $22\frac{2}{3}$. Therefore, $17 = \frac{3}{4}$ of $22\frac{2}{3}$.

NOTE. To prove this, see if $\frac{3}{4}$ of $22\frac{2}{3} = 17$.

2. $36 = \frac{4}{7}$ of $*$?
3. $32 = \frac{4}{9}$ of $*$?
4. $40 = \frac{3}{8}$ of $*$?
5. $42 = \frac{6}{7}$ of $*$?
6. $81 = \frac{9}{10}$ of $*$?
7. $27 = \frac{7}{8}$ of $*$?
8. $81 = \frac{8}{9}$ of $*$?
9. $64 = \frac{8}{5}$ of $*$?
10. $40 = \frac{5}{8}$ of $*$?
11. $\frac{4}{5}$ of $45 = \frac{6}{7}$ of $*$?
12. $3\frac{4}{5}$ times $10 = \frac{7}{8}$ of $*$?
13. $9\frac{1}{7}$ times $7 = \frac{8}{9}$ of $*$?
14. $5\frac{2}{3}$ times $9 = \frac{5}{6}$ of $*$?
15. $8\frac{3}{4}$ times $8 = \frac{9}{10}$ of $*$?
16. $8\frac{2}{3}$ times $9 = 1\frac{3}{7}$ times $*$?
17. $9\frac{1}{3}$ times $6 = 2\frac{1}{4}$ times $*$?
18. $8\frac{4}{5}$ times $10 = 1\frac{2}{7}$ times $*$?
19. $3\frac{2}{3}$ times $9 = 3\frac{1}{2}$ times $*$?
20. $5\frac{1}{2}$ times $8 = 1\frac{3}{4}$ times $*$?
21. $8\frac{2}{5}$ times $6 = 2\frac{1}{3}$ times $*$?

C. 1. If $\frac{3}{4}$ of a gallon of molasses cost 25 cents, what will 1 gallon cost?

SOLUTION. If $\frac{3}{4}$ of a gallon cost 25 cents, $\frac{1}{4}$ of a gallon will cost $\frac{1}{3}$ of 25 cents, which is $8\frac{1}{3}$ cents; and $\frac{4}{4}$ of a gallon will cost 4 times $8\frac{1}{3}$ cents, which are $33\frac{1}{3}$ cents. Therefore, 1 gallon of molasses will cost $33\frac{1}{3}$ cents, if $\frac{3}{4}$ of a gallon cost 25 cents.

How much will—

2. A yard of muslin cost, if $\frac{5}{8}$ of a yard cost 35 cents?

3. A yard of linen cost, if $\frac{4}{9}$ of a yard cost 25 cents?

4. A bushel of apples cost, if $\frac{1}{12}$ of a bushel cost 66 cents?

5. A melon cost, if $\frac{2}{3}$ of a melon cost 20 cents?

6. A pound of chocolate cost, if $\frac{3}{4}$ of a pound cost $13\frac{1}{2}$ cents?

7. A pound of sugar cost, if $4\frac{1}{2}$ pounds cost 36 cents?

8. A quart of spirit cost, if $1\frac{3}{4}$ quarts cost 50 cents?

9. $7\frac{1}{2}$ pounds of cheese cost, if $2\frac{2}{3}$ pounds cost 24 cents?

10. 3 yards of ribbon cost, if $\frac{8}{9}$ of a yard cost 48 cents?

11. $5\frac{2}{5}$ pounds of soda cost, if $1\frac{5}{8}$ pounds cost 22 cents?

12. $9\frac{3}{4}$ boxes of prunes weigh, if $4\frac{2}{7}$ boxes weigh 60 pounds?

13. David has 12 dollars, and $3\frac{2}{3}$ times his money is equal to just $\frac{4}{5}$ of Samuel's money. How many dollars has Samuel?

14. Susan is 15 years old, and $\frac{2}{3}$ of her age is 14 years less than $1\frac{1}{2}$ times Jane's age. What is Jane's age?

15. If $7\frac{2}{3}$ dozen of eggs worth 9 cents per dozen are worth as much as $\frac{7}{9}$ of a gallon of oil, what is a gallon of oil worth?

16. Edgar has money enough to buy $2\frac{1}{2}$ acres of land at $64 per acre, and $\frac{3}{10}$ of his money increased by $6 is $\frac{4}{7}$ of Alfred's money. How much money has each?

17. If $\frac{3}{7}$ of a yard of muslin at 70 cents per yard cost $\frac{5}{6}$ as much as a yard of cambric, what will a yard of cambric cost?

18. When $\frac{4}{7}$ of a yard of linen at 56 cents per yard costs $\frac{8}{9}$ as much as a yard of lawn, how much will a yard of lawn cost? How much will $\frac{7}{8}$ of a yard of lawn cost?

19. If Joseph can earn 54 cents in one day, and it takes William $\frac{8}{10}$ of a day to earn as much as Joseph can earn in $\frac{4}{5}$ of a day, how much can William earn in one day?

20. I gave 25 dollars for a cow, and $1\frac{2}{5}$ times what I gave for the cow is equal to $2\frac{1}{3}$ times what I gave for a heifer. What did I give for the heifer?

21. After giving $\frac{2}{9}$ of my money for a blank book, and $\frac{5}{9}$ of it for a grammar, I had 18 cents left. How many cents did I have at first?

22. $\frac{2}{7}$ of Mr. Ball's land is tillage, $\frac{4}{7}$ of it is pasturage, and the rest, 3 acres, is orchard. How many acres does he own?

SECTION XXV.

A. 1. What is the effect of multiplying the numerator of the fraction $\frac{3}{5}$ by 4?

ANSWER. Multiplying the numerator of the fraction $\frac{3}{5}$ by 4, gives $\frac{12}{5}$ for a result, which expresses 4 times as many parts, each of the same size as before, and is, therefore, 4 times as large. Hence, multiplying the numerator of the fraction $\frac{3}{5}$ by 4, multiplies the fraction by 4.

What is the effect of multiplying the numerator of—

2. $\frac{3}{11}$ by 2? **4.** $\frac{5}{7}$ by 3? **6.** $\frac{4}{7}$ by 5?

3. $\frac{4}{25}$ by 6? **5.** $\frac{8}{9}$ by 7? **7.** $\frac{3}{4}$ by 9?

8. What is the effect of dividing the numerator of the fraction $\frac{12}{13}$ by 6?

ANSWER. Dividing the numerator of the fraction $\frac{12}{13}$ by 6, gives $\frac{2}{13}$ for a result, which expresses $\frac{1}{6}$ as many parts, each of the same size as before, and is, therefore, $\frac{1}{6}$ as large. Hence, dividing the numerator of the fraction $\frac{12}{13}$ by 6, divides the fraction by 6.

NOTE. The principle involved in the above solutions is that "4 times $\frac{3}{5} = \frac{12}{5}$, just as 4 times 3 apples = 12 apples;" and that "$\frac{1}{6}$ of $\frac{12}{13} = \frac{2}{13}$, just as $\frac{1}{6}$ of 12 apples = 2 apples."

What is the effect of dividing the numerator of—

9. $\frac{15}{17}$ by 5? **11.** $\frac{8}{9}$ by 4? **13.** $\frac{42}{53}$ by 7?
10. $\frac{10}{11}$ by 2? **12.** $\frac{2}{5}$ by 3? **14.** $\frac{72}{89}$ by 9?

B. **1.** Which parts are larger in size, halves or fourths?

ANSWER. Halves are larger than fourths, because it takes a less number of them to equal a unit.

Which parts are larger in size—

2. Halves or thirds?
3. Fourths or eighths?
4. Thirds or ninths?
5. Halves or tenths?
6. Fourths or twelfths?
7. Fifths or twentieths?

C. **1.** Each half equals how many sixths?

FIRST FORM OF ANSWER. Each half equals $\frac{1}{2}$ of 6 sixths, which is 3 sixths. Hence, each half equals 3 sixths.

SECOND FORM OF ANSWER. Each half equals 3 sixths, for if a unit should be divided into 6 equal parts, $\frac{1}{2}$ of the unit would contain 3 of them.

2. Each half equals how many fourths?
3. Each third equals how many ninths?

4. Each fourth equals how many twelfths?
5. Each fifth equals how many tenths?
6. Each sixth equals how many eighteenths?

D. 1. What is the effect of multiplying the denominator of the fraction $\frac{3}{5}$ by 4?

ANSWER. Multiplying the denominator of the fraction $\frac{3}{5}$ by 4, gives $\frac{3}{20}$ for a result, which expresses the same number of parts each $\frac{1}{4}$ as large as before. Hence, multiplying the denominator of $\frac{3}{5}$ by 4, has divided the fraction by 4.

What is the effect of multiplying the denominator of—

2. $\frac{2}{5}$ by 5? 4. $\frac{2}{3}$ by 2? 6. $\frac{5}{8}$ by 3?
3. $\frac{3}{5}$ by 6? 5. $\frac{4}{7}$ by 3? 7. $\frac{3}{4}$ by 4?

8. What is the effect of dividing the denominator of the fraction $\frac{5}{12}$ by 2?

ANSWER. Dividing the denominator of the fraction $\frac{5}{12}$ by 2, gives $\frac{5}{6}$ for a result, which expresses the same number of parts each twice as large as before. Hence, dividing the denominator of $\frac{5}{12}$ by 2 multiples the fraction by 2.

What is the effect of dividing the denominator of—

9. $\frac{4}{9}$ by 3? 11. $\frac{7}{24}$ by 8? 13. $\frac{4}{21}$ by 7?
10. $\frac{5}{6}$ by 2? 12. $\frac{3}{25}$ by 5? 14. $\frac{11}{18}$ by 9?

E. The numerator and denominator are called TERMS of the fraction.

1. What is the effect of multiplying both terms of the fraction $\frac{2}{3}$ by 6?

ANSWER. Multiplying both terms of the fraction $\frac{2}{3}$ by 6, *gives* $\frac{12}{18}$ for a result, which expresses 6 times as many

parts, each $\frac{1}{8}$ as large as before. Hence, the value of the fraction is not altered, or $\frac{2}{3} = \frac{16}{24}$.

What is the effect of multiplying both terms of—

2. $\frac{1}{2}$ by 3?	5. $\frac{9}{10}$ by 3?	8. $\frac{4}{5}$ by 8
3. $\frac{3}{4}$ by 2?	6. $\frac{3}{8}$ by 4?	9. $\frac{2}{11}$ by 5?
4. $\frac{5}{7}$ by 4?	7. $\frac{2}{9}$ by 7?	10. $\frac{8}{9}$ by 6?

11. What is the effect of dividing both terms of the fraction $\frac{15}{25}$ by 5?

ANSWER. Dividing both terms of the fraction $\frac{15}{25}$ by 5, gives $\frac{3}{5}$ for a result, which expresses $\frac{1}{5}$ as many parts each 5 times as large as before. Hence, the value of the fraction is not altered, or $\frac{15}{25} = \frac{3}{5}$.

What is the effect of dividing both terms of—

12. $\frac{9}{12}$ by 3?	15. $\frac{8}{12}$ by 3?	18. $\frac{14}{49}$ by 7?
13. $\frac{4}{8}$ by 4?	16. $\frac{9}{15}$ by 3?	19. $\frac{35}{45}$ by 5?
14. $\frac{10}{15}$ by 5?	17. $\frac{14}{21}$ by 7?	20. $\frac{81}{99}$ by 9?

F. From the foregoing, we infer that—

1st. Multiplying the numerator multiplies the fraction, by multiplying the number of parts considered, without affecting their size.

2d. Dividing the numerator divides the fraction, by dividing the number of parts considered, without affecting their size.

3d. Multiplying the denominator divides the fraction, by dividing each part, without affecting the number of parts considered.

4th. Dividing the denominator multiplies the fraction, by multiplying each part, without affecting the number of parts considered.

5th. A fraction may be multiplied either by multiplying the numerator or by dividing the denominator.

6th. A fraction may be divided either by dividing the numerator or by multiplying the denominator.

7th. Multiplying both numerator and denominator of a fraction by the same number both multiplies and divides the fraction by that number, and, therefore, does not alter its value.

8th. Dividing both numerator and denominator of a fraction by the same number, both divides and multiplies the fraction by that number, and, therefore, does not alter its value.

SECTION XXVI.

A. A fraction is in its LOWEST TERMS when its numerator and denominator are the smallest entire numbers which will express its value.

When a fraction is in its lowest terms, both numerator and denominator are prime to each other. Hence—

A fraction may be reduced to its lowest terms by dividing both numerator and denominator by all their common factors, or by their greatest common divisor

1. Reduce $\frac{18}{24}$ to its lowest terms.

SOLUTION. Dividing both terms of $\frac{18}{24}$ by their greatest common divisor, 6, gives $\frac{3}{4}$ for a result, which expreses $\frac{1}{8}$ as many parts, each 6 times as large as before. Hence, $\frac{18}{24}$ reduced to its lowest terms, equals $\frac{3}{4}$.

The numbers by which we divide in reducing fractions to lower terms, are said to be CANCELLED. In the above solution, *we cancelled* the factor 6.

Reduce each of the following fractions to its lowest terms:—

2. $\frac{6}{12}$.	5. $\frac{12}{15}$.	8. $\frac{42}{56}$.	11. $\frac{18}{20}$.
3. $\frac{9}{15}$.	6. $\frac{16}{20}$.	9. $\frac{28}{35}$.	12. $\frac{48}{72}$.
4. $\frac{8}{24}$.	7. $\frac{36}{64}$.	10. $\frac{18}{27}$.	13. $\frac{54}{72}$.

B. A fraction is sometimes expressed by the factors of its numerator and denominator.

ILLUSTRATION. The fraction $\frac{4 \times 9}{15 \times 8}$, which may be read, "The fraction having 4 times 9 for its numerator, and 15 times 8 for its denominator," or, "The fraction 4 times 9, divided by 15 times 8."

Such fractions should be reduced to their lowest terms before multiplying their factors together.

1. Reduce $\frac{4 \times 9}{15 \times 8}$ to its lowest terms.

SOLUTION. Cancelling 4 from the 4 of the numerator and from the 8 of the denominator, gives 1 in place of the former, and 2 in place of the latter. Cancelling 3 from the 9 of the numerator, and from the 15 of the denominator, gives 3 in place of the former, and 5 in place of the latter. As no further cancellation can be made, we multiply the remaining factors together, which gives $\frac{1 \times 3}{5 \times 2} = \frac{3}{10}$.

NOTE. In Written Arithmetic, it is customary to draw a line through the numbers from which factors have been cancelled, and to write the quotients above the dividends of the numerator, and beneath those of the denominator, as illustrated below.

$$\frac{1 \times 3}{5 \times 2} = \frac{3}{10}$$

Reduce each of the following to its lowest terms:—

2. $\frac{6 \times 4}{9 \times 8}$.
3. $\frac{10 \times 9}{21 \times 5}$.
4. $\frac{12 \times 7}{35 \times 18}$.
5. $\frac{16 \times 18}{45 \times 24}$.
6. $\frac{28 \times 36}{24 \times 14}$.
7. $\frac{49 \times 25}{35 \times 35}$.
8. $\frac{2 \times 3 \times 4}{6 \times 5 \times 8}$.
9. $\frac{4 \times 7 \times 2}{7 \times 6 \times 8}$.
10. $\frac{3 \times 4 \times 6}{6 \times 8 \times 3}$.
11. $\frac{4 \times 9 \times 21}{14 \times 6 \times 3}$.
12. $\frac{15 \times 7 \times 13}{13 \times 35 \times 9}$.
13. $\frac{24 \times 18 \times 25}{36 \times 45 \times 56}$.

SECTION XXVII.

A. 1. What is the effect of multiplying the numerator of a fraction by 3, and the denominator by 4?

ANSWER. Multiplying the numerator of a fraction by 3, and the denominator by 4, gives for a result $\frac{3}{4}$ of the original fraction, for it gives 3 times as many parts, each $\frac{1}{4}$ as large as before.

What is the effect of multiplying the numerator of a fraction—

2. By 4, and the denominator by 7?
3. By 8, and the denominator by 8?
4. By 11, and the denominator by 6?
5. By 12, and the denominator by 10?
6. By 24, and the denominator by 17?

B. 1. How can $\frac{5}{8}$ of a fraction be found?

FIRST METHOD. $\frac{5}{8}$ of a fraction can be found by getting $\frac{5}{8}$ of the numerator for a new numerator, without altering the denominator, thus getting $\frac{5}{8}$ as many parts each of the *same size as before*.

SECOND METHOD. $\frac{5}{6}$ of a fraction can be found by multiplying the numerator by 5 and the denominator by 6, thus getting 5 times as many parts, each $\frac{1}{6}$ as large as before.

NOTE. The first method is identical with that of Section XXII.

Explain the methods of finding—

2. $\frac{7}{8}$ of a fraction.
3. $\frac{6}{7}$ of a fraction.
4. $\frac{3}{11}$ of a fraction.
5. $\frac{4}{13}$ of a fraction.
6. $\frac{3}{5}$ of a fraction.
7. $1\frac{3}{4}$ of a fraction.
8. $\frac{5}{12}$ of a fraction.
9. $\frac{8}{9}$ of a fraction.

C. 1. What is $\frac{5}{6}$ of $\frac{9}{10}$?

SOLUTION. $\frac{5}{6}$ of $\frac{9}{10}$, found by multiplying the numerator of $\frac{9}{10}$ by 5, and the denominator by 6, equals $\frac{9 \times 5}{10 \times 6}$, which by cancelling equals $\frac{3 \times 1}{2 \times 2} = \frac{3}{4}$. Hence, $\frac{5}{6}$ of $\frac{9}{10} = \frac{3}{4}$.

What is—

2. $\frac{7}{8}$ of $\frac{3}{8}$?
3. $\frac{5}{8}$ of $\frac{4}{10}$?
4. $\frac{4}{15}$ of $\frac{5}{8}$?
5. $\frac{3}{8}$ of $\frac{7}{8}$?
6. $\frac{3}{4}$ of $\frac{2}{3}$?
7. $\frac{24}{25}$ of $\frac{35}{36}$?
8. $\frac{36}{49}$ of $\frac{14}{32}$?
9. $\frac{2}{3}$ of $\frac{3}{4}$?
10. $\frac{8}{9}$ of $\frac{15}{16}$?
11. $\frac{9}{10}$ of $\frac{4}{5}$?
12. $\frac{7}{8}$ of $\frac{20}{21}$?
13. $\frac{4}{11}$ of $\frac{9}{10}$?

D. 1. What is $\frac{2}{5}$ of $23\frac{1}{2}$?

ABBREVIATED SOLUTION. $\frac{2}{5}$ of $23\frac{1}{2} = \frac{2}{5}$ of $20 + \frac{2}{5}$ of $3\frac{1}{2}$. But $\frac{2}{5}$ of $20 = 8$, and $\frac{2}{5}$ of $3\frac{1}{2}$ or of $\frac{7}{2} = \frac{7}{5} = 1\frac{2}{5}$, which added to $8 = 9\frac{2}{5}$. Hence, $\frac{2}{5}$ of $23\frac{1}{2} = 9\frac{2}{5}$.

What is—

2. $\frac{3}{7}$ of $22\frac{1}{4}$?
3. $\frac{5}{7}$ of $37\frac{3}{7}$?
4. $\frac{6}{11}$ of $36\frac{2}{3}$?
5. $\frac{4}{5}$ of $28\frac{1}{7}$?
6. $\frac{3}{4}$ of $49\frac{3}{4}$?
7. $\frac{5}{8}$ of $63\frac{1}{8}$?
8. $\frac{5}{9}$ of $41\frac{2}{5}$?
9. $\frac{2}{3}$ of $35\frac{2}{7}$?
10. $\frac{5}{6}$ of $58\frac{1}{2}$?

E. Since multiplying a number by 1 gives the number itself for a product, multiplying a number by $\frac{2}{3}$ of 1, or by $\frac{2}{3}$, must give $\frac{2}{3}$ of the number for a product; multiplying a number by $\frac{5}{6}$ must give $\frac{5}{6}$ of the number for a product, and generally—

To multiply any number by a fraction is merely to get such a part of it as the given fraction denotes.

1. What is the product of $\frac{9}{14} \times \frac{28}{33}$?

FIRST SOLUTION. $\frac{9}{14}$ times $\frac{28}{33} = \frac{9}{14}$ of $\frac{28}{33} = \frac{9 \times 28}{14 \times 33} = \frac{6}{11}$.

SECOND SOLUTION. $\frac{9}{14}$ multiplied by $\frac{28}{33} = \frac{28}{33}$ of $\frac{9}{14} = \frac{28 \times 9}{33 \times 14} = \frac{6}{11}$.

NOTE. The slight difference between the first and second solution results from the different reading of the sign of multiplication. We recommend the first as being the most simple.

2. What is the product of $4\frac{1}{2} \times 2\frac{5}{6}$?

SOLUTION. Since $4\frac{1}{2} = \frac{9}{2}$, and $2\frac{5}{6} = \frac{17}{6}$, the product of $4\frac{1}{2} \times 2\frac{5}{6} = \frac{9}{2} \times \frac{17}{6} = \frac{9 \times 17}{2 \times 6} = \frac{51}{4} = 12\frac{3}{4}$.

What is the product of—

3. $\frac{3}{5} \times \frac{11}{12}$?
4. $\frac{11}{12} \times \frac{3}{5}$?
5. $\frac{6}{7} \times \frac{7}{12}$?
6. $\frac{20}{27} \times \frac{63}{80}$?
7. $\frac{32}{81} \times \frac{54}{64}$?
8. $\frac{42}{55} \times \frac{33}{28}$?
9. $\frac{5}{6} \times \frac{3}{8} \times \frac{16}{15}$?
10. $\frac{3}{4} \times \frac{5}{6} \times \frac{8}{10}$?
11. $\frac{4}{7} \times \frac{7}{8} \times \frac{8}{9} \times \frac{9}{16}$?

F. How much will—

1. $\frac{3}{4}$ of a yd. of cloth cost, at $\frac{5}{6}$ of a dollar per yd.?
2. $\frac{2}{3}$ of a qt. of filberts cost, at $\frac{2}{3}$ of a dime per qt.?

3. $\frac{5}{7}$ of a lb. of tea cost, at $\frac{4}{5}$ of a dollar per lb.?
4. $\frac{8}{9}$ of a day's labor cost, at $\frac{2}{3}$ of a dollar per day?
5. $\frac{2}{3}$ of a bag of grain cost, at $\frac{7}{9}$ of a dollar per bag?
6. $2\frac{1}{2}$ yd. of silk cost, at $\frac{9}{10}$ of a dollar per yd.?
7. $3\frac{5}{9}$ boxes of soap cost, at $\$3\frac{3}{8}$ per box?
8. 5 pine-apples cost, if 6 cost $\frac{3}{4}$ of a dollar?
9. 10 lb. of coffee cost, if 5 lb. cost $\frac{3}{7}$ of a dollar?

10. Rufus had $\frac{7}{8}$ of a dollar, and spent $\frac{2}{3}$ of what he had. What part of a dollar did he spend?

11. If a man can hoe $\frac{8}{9}$ of an acre per day, how much can he hoe in $2\frac{1}{4}$ days?

12. If $\frac{2}{3}$ of a yard of velvet cost $5\frac{1}{4}$ dollars, what will 4 yards cost?

SOLUTION. If $\frac{2}{3}$ of a yard of velvet cost $5\frac{1}{4}$ dollars, $\frac{1}{3}$ of a yard will cost $\frac{1}{2}$ of $5\frac{1}{4}$ dollars $= \frac{1}{2}$ of 4 dollars, which is 2 dollars, plus $\frac{1}{2}$ of $1\frac{1}{4}$ dollars, or of $\frac{5}{4}$ of a dollar, which is $\frac{5}{8}$ of a dollar. Therefore, $\frac{1}{3}$ of a yard will cost $2\frac{5}{8}$ dollars. If $\frac{1}{3}$ of a yard costs $2\frac{5}{8}$ dollars, 3 thirds of a yard will cost 3 times $2\frac{5}{8}$ dollars, which are $7\frac{7}{8}$ dollars, and 4 yards will cost 4 times $7\frac{7}{8}$ dollars, which are $31\frac{1}{2}$ dollars. Hence, 4 yards of velvet will cost $31\frac{1}{2}$ dollars if $\frac{2}{3}$ of a yard cost $5\frac{1}{4}$ dollars.

How much will—

13. An acre cost, if $\frac{5}{8}$ of an acre cost $\$30\frac{1}{2}$?
14. A cask of oil cost, if $\frac{7}{9}$ of a cask cost $\$64\frac{2}{3}$?
15. A cord of wood cost, if $2\frac{1}{2}$ cords cost $\$18\frac{3}{4}$?
16. A man earn in 1 day, if he earns $\$5\frac{3}{5}$ in $2\frac{1}{3}$ days?
17. $2\frac{2}{3}$ days' labor cost, at $2\frac{2}{3}$ dollars per day?
18. 1 day's labor cost, if $2\frac{2}{3}$ days' labor cost $\$7\frac{1}{9}$?

19. How many pages will Albert read in $2\frac{1}{4}$ hours, if he reads $13\frac{1}{3}$ pages in 4 hours?

20. How many pages will Albert read in 4 hours, if he reads $7\frac{1}{2}$ pages in $2\frac{1}{4}$ hours?

21. If $\frac{3}{4}$ of a yard of linen is given for $\frac{3}{5}$ of a yard of silk, worth $\frac{5}{6}$ of a dollar per yard, what ought the linen to be worth per yard?

22. If $\frac{3}{5}$ of a yard of silk is given for $\frac{3}{4}$ of a yard of linen, worth $\frac{2}{3}$ of a dollar per yard, what ought the silk to be worth per yard?

23. I have $\$2\frac{1}{2}$, and $\frac{2}{3}$ of my money equals $\frac{5}{8}$ of Edmund's. How many dollars has Edmund?

24. My father gave me $\frac{3}{4}$ of a pine-apple, and $\frac{5}{6}$ of what he gave me equals $\frac{15}{16}$ of what he gave to my sister. What part of a pine-apple did he give to my sister?

25. What fractional part of 1 rod is 4 yd. 2 ft. $1\frac{5}{7}$ in.?

SOLUTION. Since 1 in. $= \frac{1}{12}$ of a foot, $1\frac{5}{7}$ in. or $\frac{12}{7}$ of an inch must equal $\frac{12}{7}$ of $\frac{1}{12}$ of a ft. $= \frac{1}{7}$ ft., to which, adding the 2 ft., gives $2\frac{1}{7}$ ft., or $\frac{15}{7}$ ft. Since 1 ft. $= \frac{1}{3}$ of a yd., $\frac{15}{7}$ of a ft. must equal $\frac{15}{7}$ of $\frac{1}{3}$ of a yd. $= \frac{5}{7}$ yd., to which, adding the 4 yd., gives $4\frac{5}{7}$ yd. $= \frac{33}{7}$ yd. Since 1 yd. $= \frac{2}{11}$ of a rd., $\frac{33}{7}$ of a yd. must equal $\frac{33}{7}$ of $\frac{2}{11}$ rd. $= \frac{6}{7}$ rd. Hence, 4 yd. 2 ft. $1\frac{5}{7}$ in. $= \frac{6}{7}$ of a rod.

NOTE. Prove by Solution to 24th problem, page 84.

What fractional part—

26. Of 1 bu. is 3 pk. 1 qt. $1\frac{1}{8}$ pt.?

27. Of 1 gal. is 2 qt. 1 pt. $2\frac{6}{7}$ gil.?

28. Of 1 lb. is 8 oz. $14\frac{2}{3}$ dr.?

29. Of 1 wk. is 5 da. 10 h. 40 m.?

30. Of 1 £ is 2 s. 10 d. $1\frac{1}{7}$ qr.?

31. Of 1 ℔ is 9 ℥ 4 ʒ 2 ℈ 8 gr.?

32. Of 1 lb. is 4 oz. 5 dwt. $17\frac{1}{7}$ gr.?

33. Of 1 T. is 2 cwt. 0 qr. 22 lb. 3 oz. $8\frac{8}{9}$ oz.

SECTION XXVIII.

A. 1. 1 = how many times $\frac{1}{5}$?

SOLUTION. 1 = $\frac{5}{5}$, and $\frac{5}{5}$ = 5 times $\frac{1}{5}$. Hence, 1 = 5 times $\frac{1}{5}$.

2. 1 = * times $\frac{1}{2}$?
3. 1 = * times $\frac{1}{9}$?
4. 1 = * times $\frac{1}{12}$?
5. 1 = * times $\frac{1}{20}$?
6. 1 ÷ $\frac{1}{4}$?
7. 1 ÷ $\frac{1}{7}$?
8. 1 ÷ $\frac{1}{8}$?
9. 1 ÷ $\frac{1}{6}$?

B. Since 1 = 2 times $\frac{1}{2}$, 3 times $\frac{1}{3}$, &c., it follows that any number must contain $\frac{1}{2}$, 2 times as many times as it contains 1; that it must contain $\frac{1}{3}$, 3 times as many times as it contains 1, &c., &c.

Again—Since the quotient of any number divided by 1 equals the number itself, the quotient of any number divided by $\frac{1}{5}$ must equal 5 times that number; the quotient of any number divided by $\frac{1}{6}$ must equal 6 times that number, &c., &c.

Hence, the quotient of any number divided by a fraction having 1 for a numerator equals the product of that number multiplied by the denominator of the fraction.

1. What is the quotient of 8 ÷ $\frac{1}{3}$?

SOLUTION. Since the quotient of 8 ÷ 1 = 8, the quotient of 8 ÷ $\frac{1}{3}$ must equal 3 times 8 or 24. Hence, the quotient of 8 ÷ $\frac{1}{3}$ = 24.

2. What is the quotient of $\frac{5}{8}$ ÷ $\frac{1}{7}$?

SOLUTION. Since the quotient of $\frac{5}{8}$ divided by 1 = $\frac{5}{8}$,

the quotient of $\frac{5}{8}$ divided by $\frac{1}{7}$ must equal 7 times $\frac{5}{8}$, which are $\frac{35}{8} = 4\frac{3}{8}$. Hence, the quotient of $\frac{5}{8} \div \frac{1}{7} = 4\frac{3}{8}$.

What is the quotient of—

3.	$8 \div \frac{1}{7}$?	**7.**	$12 \div \frac{1}{5}$?	**11.**	$\frac{8}{9} \div \frac{1}{10}$?
4.	$2 \div \frac{1}{9}$?	**8.**	$9 \div \frac{1}{8}$?	**12.**	$\frac{4}{11} \div \frac{1}{9}$?
5.	$10 \div \frac{1}{4}$?	**9.**	$\frac{5}{7} \div \frac{1}{6}$?	**13.**	$\frac{3}{8} \div \frac{1}{5}$?
6.	$4 \div \frac{1}{3}$?	**10.**	$\frac{3}{5} \div \frac{1}{6}$?	**14.**	$\frac{5}{12} \div \frac{1}{4}$?

C. **1.** What is the quotient of $4 \div \frac{3}{7}$?

SOLUTION. Since the quotient of 4 divided by $1 = 4$, the quotient of 4 divided by $\frac{1}{7}$ must equal 7 times 4, and the quotient of 4 divided by $\frac{3}{7}$ must equal $\frac{1}{3}$ of 7 times 4, $= \frac{7}{3}$ of 4, which is $9\frac{1}{3}$. Hence, the quotient of $4 \div \frac{3}{7} = 9\frac{1}{3}$.

What is the quotient of—

2.	$7 \div \frac{2}{3}$?	**6.**	$1 \div \frac{7}{8}$?	**10.**	$6 \div \frac{2}{9}$?
3.	$5 \div \frac{4}{5}$?	**7.**	$4 \div \frac{5}{12}$?	**11.**	$1 \div \frac{3}{8}$?
4.	$8 \div \frac{3}{4}$?	**8.**	$8 \div \frac{5}{11}$?	**12.**	$1 \div \frac{2}{5}$?
5.	$1 \div \frac{2}{9}$?	**9.**	$2 \div \frac{7}{12}$?	**13.**	$9 \div \frac{4}{7}$?

D. **1.** What is the quotient of $\frac{3}{4} \div \frac{6}{7}$?

SOLUTION. Since the quotient of $\frac{3}{4}$ divided by $1 = \frac{3}{4}$, the quotient of $\frac{3}{4} \div \frac{1}{7}$ must equal 7 times $\frac{3}{4}$, and the quotient of $\frac{3}{4}$ divided by $\frac{6}{7}$ must equal $\frac{1}{6}$ of 7 times $\frac{3}{4}$, $= \frac{7}{6}$ of $\frac{3}{4}$, which equals $\frac{7}{8}$. Hence, the quotient of $\frac{3}{4} \div \frac{6}{7} = \frac{7}{8}$.

What is the quotient of—

2.	$\frac{3}{8} \div \frac{9}{10}$?	**6.**	$\frac{2}{7} \div \frac{5}{11}$?	**10.**	$\frac{9}{10} \div \frac{3}{8}$?
3.	$\frac{1}{3} \div \frac{6}{5}$?	**7.**	$\frac{4}{7} \div \frac{5}{14}$?	**11.**	$\frac{5}{8} \div \frac{2}{7}$?
4.	$\frac{2}{7} \div \frac{4}{11}$?	**8.**	$\frac{8}{9} \div \frac{4}{5}$?	**12.**	$\frac{8}{9} \div \frac{8}{11}$?
5.	$\frac{5}{9} \div \frac{3}{8}$?	**9.**	$9\frac{1}{7} \div \frac{3}{8}$?	**13.**	$\frac{2}{7} \div \frac{5}{8}$?

E. Since the quotient of a number divided by $\frac{2}{3} = \frac{1}{2}$ of 3 times the number $= \frac{3}{2}$ of the number; and, since the quotient of a number divided by $\frac{4}{9} = \frac{1}{4}$ of 9 times the number $= \frac{9}{4}$ of the number, &c., it follows that—

The quotient of a number divided by a fraction equals the product of that number multiplied by the fraction inverted.

1. What is the quotient of $1\frac{7}{9} \div 2\frac{2}{3}$?

SOLUTION. $1\frac{7}{9} = \frac{16}{9}$; $2\frac{2}{3} = \frac{8}{3}$; and since the quotient of a number divided by a fraction equals the product of that number multiplied by the fraction inverted, the quotient of $\frac{16}{9} \div \frac{8}{3} = \frac{3}{8}$ of $\frac{16}{9} = \frac{2}{3}$. Hence, the quotient of $1\frac{7}{9} \div 2\frac{2}{3} = \frac{2}{3}$.

What is the quotient of—

2. $\frac{4}{7} \div 1\frac{1}{3}$?
3. $\frac{3}{5} \div 7\frac{1}{2}$?
4. $5\frac{1}{4} \div 4\frac{2}{3}$?
5. $2\frac{1}{4} \div 4\frac{1}{2}$?
6. $\frac{15}{16} \div 3\frac{1}{4}$?
7. $9\frac{3}{5} \div 1\frac{5}{7}$?
8. $8\frac{1}{3} \div 4\frac{1}{8}$?
9. $\frac{9}{10} \div \frac{5}{6}$?
10. $8\frac{1}{4} \div 5\frac{3}{8}$?

F. Examples in division of fractions sometimes appear in the form of fractions. They are then called COMPLEX FRACTIONS, the numerator being the dividend, and the denominator the divisor.

ILLUSTRATION. $\frac{4\frac{2}{3}}{3\frac{1}{5}}$, which equals $4\frac{2}{3} \div 3\frac{1}{5}$?

A complex fraction, then, has a fraction in one or both its terms.

Complex fractions may be explained after the following model: $\frac{4\frac{2}{3}}{3\frac{1}{5}}$ expresses the value of $4\frac{2}{3}$ equal parts of such kind that $3\frac{1}{5}$ of them will equal a unit.

Complex fractions may be reduced to simple ones by merely performing the indicated division.

Thus: $\frac{4\frac{2}{3}}{3\frac{1}{5}} = 4\frac{2}{3} \div 3\frac{1}{5} = \frac{14}{3} \div \frac{16}{5} = \frac{5}{16}$ of $\frac{14}{3} = \frac{35}{24} = 1\frac{11}{24}$.

Reduce each of the following to simple fractions:—

1. $\frac{1\frac{2}{3}}{2\frac{1}{2}}$.
2. $\frac{3\frac{3}{4}}{2\frac{1}{7}}$.
3. $\frac{3\frac{1}{2}}{4\frac{2}{3}}$.
4. $\frac{5\frac{1}{7}}{8\frac{3}{4}}$.
5. $\frac{4\frac{1}{5}}{6\frac{3}{10}}$.
6. $\frac{3\frac{8}{9}}{1\frac{1}{6}}$.

G. 1. How many melons at $\frac{3}{4}$ of a dime each can be bought for 5 dimes?

SOLUTION. Since 1 melon can be bought for $\frac{3}{4}$ of a dime, as many melons can be bought for 5 dimes as there are times $\frac{3}{4}$ in 5, which are $\frac{4}{3}$ of 5 times $= \frac{20}{3}$ times $= 6\frac{2}{3}$ times. Hence, $6\frac{2}{3}$ melons at $\frac{3}{4}$ of a dime each can be bought for 5 dimes.

2. How many bushels of corn at $\frac{5}{8}$ of a dollar per bushel can be bought for 7 dollars?

3. How many hours will it take a scholar who learns $\frac{6}{7}$ of a page per hour to learn 3 pages?

4. When tea is worth $\frac{4}{9}$ of a dollar per lb., how many pounds can be bought for $5?

5. If a man can walk $\frac{5}{12}$ of a furlong in a minute, in how many minutes can he walk $\frac{7}{8}$ of a furlong?

6. If a man can gather $\frac{2}{9}$ of the apples on a tree in an hour, in how many hours can he gather $\frac{7}{12}$ of them?

7. How many flower-beds, each containing $\frac{1}{14}$ of a rood, can be made from $\frac{2}{7}$ of a rood of land?

8. A man who had $5, gave $\frac{2}{3}$ of his money for grass *seed at $2\frac{1}{2}$ per bushel.* How many bushels did he buy?

9. How many pounds of pearlash at $\frac{9}{10}$ of a dime per lb. can be bought for $1\frac{3}{5}$ lb. of chocolate at $3\frac{3}{4}$ dimes per lb.?

10. If Josephine can learn $\frac{7}{9}$ of her lesson in an hour, how many hours will it take her to learn the whole of it?

11. Albert had a cord 28 feet long, which he cut into pieces each $2\frac{2}{3}$ feet long. How many pieces did he obtain?

12. I had $9, and invested $\frac{3}{4}$ of my money in cloth at $\$1\frac{1}{2}$ per yd., and the rest of it in cloth at $\$1\frac{1}{5}$ per yd. How many yards of each kind did I buy?

13 How many bushels of potatoes at $\frac{3}{8}$ of a dollar per bushel can be bought for 9 bushels of corn at $\frac{1}{2}$ of a dollar per bushel?

14. How many bottles each holding $\frac{4}{5}$ of a quart, can be filled from $\frac{7}{8}$ of a gallon of wine?

15. A farmer had $2\frac{3}{7}$ tons of hay in one stack, and $3\frac{4}{7}$ tons in another. He sold it in loads each weighing $1\frac{3}{8}$ tons. How many loads were there?

16. How many tiles $\frac{3}{4}$ of a foot long and $\frac{1}{2}$ of a foot wide will it take to cover 18 sq. ft. of surface?

17. How many square yards are there in a floor 12 ft. long and 9 ft. wide, and how many yards of carpeting $\frac{3}{4}$ of a yard wide will it take to cover it?

18. When $1 is received for $\frac{2}{3}$ of a sq. ft. of land, how many dollars will be received for a strip 16 feet long, and $\frac{5}{12}$ of a foot wide?

19. How many yards of paper hangings $\frac{2}{5}$ of a yard wide will it take to paper a wall 15 ft. long and 6 ft. high, and how much will it cost at $\frac{3}{4}$ of a dime per yard?

20. I bought enough carpeting $\frac{3}{4}$ of a yard wide to cover a floor 15 ft. long and 12 ft. wide. How many yards did I buy, and how much did it cost at $\$1\frac{1}{3}$ per yard?

SECTION XXIX.

A. 1. $\frac{3}{4}$ = how many twelfths?

SOLUTION. Since $1 = \frac{12}{12}$, $\frac{3}{4}$ of 1 must equal $\frac{3}{4}$ of $\frac{12}{12}$, which are $\frac{9}{12}$. Hence, $\frac{3}{4} = \frac{9}{12}$.

In a similar manner reduce—

2. $\frac{2}{3}$ to sixths.
3. $\frac{1}{2}$ to eighths.
4. $\frac{4}{5}$ to tenths.
5. $\frac{3}{4}$ to twentieths.
6. $\frac{5}{9}$ to forty-fifths.
7. $\frac{5}{6}$ and $\frac{4}{9}$ to thirty-sixths.
8. $\frac{3}{4}$ and $\frac{3}{10}$ to fortieths.
9. $\frac{7}{12}$ and $\frac{3}{8}$ to twenty-fourths.
10. $\frac{9}{10}$ and $\frac{7}{30}$ to sixtieths.
11. $\frac{3}{8}$ and $\frac{4}{9}$ to fifty-sixths.

Now solve the above questions by the following form :—

SOLUTION TO PROBLEM FIRST. Since the required denominator, 12, is 3 times the given denominator, 4, we multiply both terms of $\frac{3}{4}$ by 3, which gives $\frac{3}{4} = \frac{9}{12}$.

B. 1. Fractions have a COMMON DENOMINATOR when they have the same denominator.

ILLUSTRATION. $\frac{5}{9}$, $\frac{8}{9}$, and $\frac{2}{9}$ have the common denominator 9, but $\frac{3}{4}$ and $\frac{1}{3}$ do not have a common denominator.

2. Fractions having different denominators can be reduced to a COMMON DENOMINATOR, *i. e.*, to equivalent fractions having a common denominator.

3. In reducing fractions to a common denominator—

1st. Select a convenient number for the common denominator.

2d. Reduce the given fractions by one of the methods *explained in A.*

4. It will usually be most convenient to select the least common multiple of the denominators of the given fractions for a common denominator.

C. 1. Reduce $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$ to a common denominator.

PARTIAL SOLUTION. We first find the least common multiple of the given denominators 6, 8, and 12. It is 24, which we therefore select for the common denominator. The problem is now equivalent to the following: "Reduce $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$ to twenty-fourths."

Reduce to a common denominator—

2. $\frac{1}{2}$ and $\frac{1}{4}$.
3. $\frac{2}{3}$ and $\frac{1}{2}$.
4. $\frac{3}{4}$ and $\frac{3}{7}$.
5. $\frac{5}{6}$ and $\frac{3}{8}$.
6. $\frac{7}{9}$, $\frac{1}{6}$, and $\frac{11}{12}$.
7. $\frac{9}{10}$, $\frac{7}{15}$, and $\frac{2}{3}$.
8. $\frac{3}{7}$, $\frac{2}{5}$, and $\frac{9}{14}$.
9. $\frac{4}{9}$, $\frac{7}{8}$, and $\frac{5}{12}$.
10. $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{1}{8}$.
11. $\frac{3}{7}$, $\frac{1}{2}$, and $\frac{2}{3}$.
12. $\frac{2}{5}$, $\frac{3}{4}$, and $\frac{7}{8}$.
13. $\frac{9}{10}$, $\frac{7}{15}$, and $\frac{1}{2}$.
14. $\frac{5}{6}$, $\frac{7}{9}$, and $\frac{3}{4}$.
15. $\frac{3}{7}$, $\frac{5}{14}$, and $\frac{1}{2}$.
16. $\frac{8}{9}$, $\frac{7}{12}$, $\frac{11}{18}$, and $\frac{3}{4}$.
17. $\frac{1}{4}$, $\frac{13}{20}$, $\frac{3}{5}$, and $\frac{9}{15}$.
18. $\frac{2}{9}$, $\frac{1}{6}$, $\frac{1}{2}$, and $\frac{5}{18}$.
19. $\frac{3}{10}$, $\frac{7}{20}$, $\frac{1}{4}$, and $\frac{2}{5}$.

SECTION XXX.

A. In order that fractions may be added or subtracted, they must be simple fractions, and have a common denominator. Hence—

Complex and compound fractions must be reduced to simple fractions, and simple fractions to a common denominator, before they can be added or subtracted.

1. What is the sum of $\frac{8}{9}$ of $\frac{3}{4} + \frac{3\frac{1}{2}}{4\frac{1}{5}} + \frac{5}{8} + \frac{7}{12}$.

ABBREVIATED SOLUTION. $\frac{8}{9}$ of $\frac{3}{4} = \frac{2}{3}$, and $\frac{3\frac{1}{2}}{4\frac{1}{5}} = \frac{5}{8}$. Hence, the problem becomes $\frac{2}{3} + \frac{5}{8} + \frac{5}{8} + \frac{7}{12}$. Selecting 24 as the common denominator, and reducing the fractions to twenty-fourths, as explained in Section XXIX., A, gives $\frac{2}{3} + \frac{5}{8} + \frac{5}{8} + \frac{7}{12} = \frac{16}{24} + \frac{20}{24} + \frac{15}{24} + \frac{14}{24} = 2\frac{17}{24}$.

2. What is the value of $\frac{4\frac{1}{2}}{5\frac{1}{7}} - \frac{15}{22}$ of $\frac{11}{18}$?

ABBREVIATED SOLUTION. $\frac{4\frac{1}{2}}{5\frac{1}{7}} = \frac{7}{8}$; $\frac{15}{22}$ of $\frac{11}{18} = \frac{5}{12}$; $\frac{7}{8} - \frac{5}{12} = \frac{21}{24} - \frac{10}{24} = \frac{11}{24}$. Hence, $\frac{4\frac{1}{2}}{5\frac{1}{7}} - \frac{15}{22}$ of $\frac{11}{18} = \frac{11}{24}$.

3. Add the fractions in each of the examples under C, Section XXIX.

4. Find the difference of the fractions in the second, third, fourth, and fifth examples under C, Section XXIX.

5. In each example following the fifth under C, Section XXIX., subtract the last fraction from the sum of the others.

What is the value of—

6. $\frac{4}{5} + \frac{2}{3}$?
7. $2\frac{1}{2} + 3\frac{1}{4}$?
8. $3\frac{2}{5} + 4\frac{5}{6}$?
9. $7\frac{3}{4} + 2\frac{2}{3}$?
10. $4\frac{5}{8} + 7\frac{1}{2}$?
11. $5\frac{1}{2} + 4\frac{2}{9}$?
12. $6\frac{3}{4} - 2\frac{2}{3}$?
13. $7\frac{1}{2} - 2\frac{1}{3}$?
14. $5\frac{4}{9} + 3\frac{3}{4}$?
15. $8\frac{2}{3} - 3\frac{4}{5}$?
16. $9\frac{7}{8} - 3\frac{5}{6}$?
17. $4\frac{1}{2} - 2\frac{21}{31}$?
18. $5\frac{6}{7} + 2\frac{3}{5}$?
19. $7\frac{1}{8} - 4\frac{3}{4}$?

What is the value of—

20. $9\frac{1}{4} + 6\frac{1}{2}$?

21. $\frac{7}{8} + \frac{3}{4} + \frac{5}{6}$?

22. $\frac{2}{3} + \frac{4}{5} + \frac{1}{2}$?

23. $\frac{7}{9} + \frac{1}{13} + \frac{2}{3}$?

24. $\frac{8}{15} + \frac{7}{10} + \frac{2}{3} + \frac{1}{6}$?

25. $\frac{1}{2}$ of $\frac{3}{4} + \frac{4}{5}$?

26. $\frac{5}{6}$ of $\frac{9}{11} + \frac{13}{22}$?

27. $3\frac{1}{4} + 8\frac{5}{9} - 6\frac{7}{12}$?

28. $4\frac{3}{5} + 6\frac{4}{9} - 7\frac{8}{15}$?

29. $\frac{3}{8}$ of $\frac{14}{15} + \frac{2}{3}$ of $\frac{1}{4}$?

30. $\frac{2\frac{1}{2}}{3\frac{1}{5}} + \frac{5}{6}$ of $\frac{9}{10}$?

31. $\frac{3}{5} + \frac{1\frac{2}{3}}{2\frac{6}{7}} + \frac{1}{2}$?

32. $\frac{1\frac{1}{2}}{3} + \frac{2\frac{2}{3}}{5\frac{1}{3}} + \frac{2\frac{1}{2}}{5}$?

33. $\frac{2}{7}$ of $\frac{8}{9}$ of $\frac{2\frac{1}{2}}{3\frac{1}{2}} + \frac{4}{9}$?

34. $8\frac{1}{2} + 3\frac{4}{5} + 7\frac{1}{3}$?

35. $\frac{3\frac{1}{2}}{8\frac{2}{5}} + \frac{4}{9}$ of $\frac{3}{8}$?

B. 1. Rufus bought a slate for $\frac{1}{4}$ of a dollar, a writing-book for $\frac{1}{8}$ of a dollar, a geography for $\frac{1}{2}$ of a dollar, and an atlas for $\frac{7}{16}$ of a dollar. What was the cost of the whole?

2. If Edward spends $1\frac{1}{3}$ hours in studying history, $1\frac{3}{4}$ hours in studying geography, and $1\frac{5}{6}$ hours in studying grammar, how many hours does he spend in studying?

3. A man gave $\frac{1}{4}$ of a pine-apple to Sarah, $\frac{1}{5}$ of it to Jane, $\frac{1}{10}$ to Susan, $\frac{3}{20}$ to Maria, and the rest to Emma. What part of it did he give to Emma?

4. I spent $\frac{2}{7}$ of my money for land, $\frac{5}{14}$ of it for buildings, and put the rest at interest. What part of it did I put at interest?

5. I bought an umbrella for $\$1\frac{3}{8}$, and a pair of shoes for $\$2\frac{5}{8}$. How much did both cost? How much more did the shoes cost than the umbrella?

6. A farmer sold $6\frac{3}{4}$ tons of hay, and then had $8\frac{5}{9}$ tons left. How many tons had he at first?

7. A man paid $2\frac{4}{5}$ for corn, $3\frac{7}{10}$ for wheat, and enough to make up $10\frac{1}{2}$ for rye. How much did he pay for rye?

8. I bought $\frac{3}{4}$ of a yard of silk velvet at $7 per yard, and $\frac{4}{5}$ of a yard of satin at $6 per yard. What was the cost of the whole?

9. I gave 4 yards of cloth at $3\frac{2}{9}$ per yard, and the balance in money, for 4 yards of cloth at $5\frac{2}{3}$ per yard. How much money did I pay?

10. This morning I had 7 quarter-eagles, but I have since paid for 8 books at $1\frac{3}{8}$ each. How much money have I left?

11. A farmer gathered $7\frac{1}{2}$ bbl. of russets, $1\frac{1}{8}$ bbl. of greenings more than of russets, and $6\frac{3}{4}$ bbl. of baldwins less than of russets and greenings together. How many barrels of each kind did he gather? How many of all?

12. Arthur has $4\frac{3}{4}$, Richard has $2\frac{5}{12}$ less than $1\frac{1}{2}$ times as much money as Arthur, and Edwin has $\frac{2}{3}$ as much as Arthur and Richard together. How much money has Edwin?

13. After spending $\frac{1}{2}$ and $\frac{1}{3}$ of his money, George has 7 dollars left. What part of his original money did he have left? How many dollars did he have at first?

14. $\frac{1}{3}$ of my life has been spent in Boston, $\frac{1}{8}$ of it has been spent in New York, and the rest, $7\frac{1}{2}$ years, has been spent in Washington. What is my age?

15. A farmer has $\frac{1}{2}$ of his sheep in one pasture, $\frac{1}{5}$ of them in another, and the rest, 111 sheep, in another. How many sheep has he, and how many in each pasture?

16. $\frac{2}{7}$ of a certain number plus $\frac{1}{3}$ of that number, plus $\frac{1}{21}$ of that number, plus 9, equals the whole of the number. What is the number?

17. If a drover sells $\frac{1}{3}$ of his sheep to one man, and $\frac{1}{4}$ of

them to another, he will sell 6 more to the first man than to the second. How many sheep has he?

18. James says that $\frac{1}{2}$ of his age exceeds $\frac{1}{3}$ of it by $1\frac{1}{2}$ years; to which George replies, "Then you are only $\frac{5}{6}$ as old as I am." What is the age of each boy?

19. $3\frac{1}{2}$ times a certain number exceeds 2 times the number by $4\frac{1}{2}$. What is the number?

20. If I could sell my cow for 13 dollars more than 3 times what she cost me, I should receive $100 for her. How much did she cost me?

21. $\frac{2}{9}$ of the trees in a certain orchard bear peaches, $\frac{1}{6}$ bear cherries, $\frac{1}{3}$ bear apples, and the rest bear pears. Now, if there are 7 more apple trees than peach trees, how many trees are there in the orchard, and how many of each kind?

22. What number is that to which, if $2\frac{2}{3}$ be added, the sum will be $1\frac{5}{6}$ less than $3\frac{3}{4}$ times the number?

23. What number is that to which, if its $\frac{1}{2}$ and $\frac{1}{8}$ part be added, the result will be 77?

24. The difference between $\frac{1}{3}$ of $\frac{2}{5}$ of a certain number, and $\frac{1}{2}$ of $1\frac{2}{3}$ times the same number equals the sum of $1\frac{1}{4} + 1\frac{1}{2}\frac{1}{0}$. What is the number?

25. If $2\frac{1}{2} + 3\frac{1}{6} + 4\frac{2}{3} = \frac{2}{9}$ of a certain number, what is $\frac{2}{5}$ of that number?

26. If I should receive as much money as I now have, and $\frac{1}{2}$ as much more, I should have $40. How much money have I?

27. William's father gave him a certain sum, his mother gave him $\frac{2}{3}$ as much as his father, and his uncle gave him 10 cents more than his father and mother together, when he had 4 times as much as his father gave him. How many cents did each give him, and how many did all?

SECTION XXXI.

A. The term PER CENT, as used in arithmetic, means the same as one hundredths.

ILLUSTRATION. 6 per cent $= \frac{6}{100}$; 7 per cent $= \frac{7}{100}$; $\frac{1}{2}$ per cent means $\frac{1}{2}$ of $\frac{1}{100}$, &c.

From this it follows that 1 = 100 per cent, or that anything equals 100 per cent of itself.

1. $\frac{2}{3}$ = how many per cent?

SOLUTION. Since 1 = 100 per cent, $\frac{2}{3}$ must equal $\frac{2}{3}$ of 100 per cent, which is $66\frac{2}{3}$ per cent.

How many per cent are equal—

2. To $\frac{1}{2}$?
3. To $\frac{1}{4}$?
4. To $\frac{1}{5}$?
5. To $\frac{1}{8}$?
6. To $\frac{3}{4}$?
7. To $\frac{5}{6}$?
8. To $\frac{3}{8}$?
9. To $\frac{7}{10}$?
10. To $\frac{4}{5}$?
11. To $\frac{5}{12}$?
12. To $\frac{2}{7}$?
13. To $\frac{4}{9}$?

B. 1. What common fraction is equal to 6 per cent?

ANSWER. 6 per cent $= \frac{6}{100}$, which reduced to its lowest terms equals $\frac{3}{50}$. Hence, 6 per cent $= \frac{3}{50}$.

What common fraction is equal to—

2. 5 per cent?
3. 8 per cent?
4. $12\frac{1}{2}$ per cent?
5. $66\frac{2}{3}$ per cent?
6. 25 per cent?
7. *20 per cent?*
8. 50 per cent?
9. 75 per cent?
10. $33\frac{1}{3}$ per cent?
11. 40 per cent?
12. 15 per cent?
13. $16\frac{2}{3}$ per cent?

C. One per cent of 1 dollar or of 100 cents is 1 cent; hence, 1 per cent of any number of dollars is as many cents as there are dollars.

ILLUSTRATION. 1 per cent of $5 is 5 cents = $.05; 1 per cent of $96 is 96 cents = $.96; 1 per cent of $728 is 728 cents = $7.28, &c.

What is 1 per cent—

1. Of $7?
2. Of $26?
3. Of $827?
4. Of $594?
5. Of $329?
6. Of $764?

D. One per cent of any number may be found by removing the decimal point two places to the left.

1. What is 1 per cent of $327.56?

ANSWER. 1 per cent of $327.56 is $3.2756, or 3 dollars, 27 cents, $5\frac{6}{10}$ mills.

What is 1 per cent—

2. Of $837.95?
3. Of $628.70?
4. Of $53.98?
5. Of $224.16?
6. Of $1784.97?
7. Of $4258.63?

E. 1. What is 7 per cent of $242?

SOLUTION. 1 per cent of $242 = $2.42, and 7 per cent of $242 must equal 7 times $2.42, which equal $16.94.

What is—

2. 3 per cent of $45?
3. 9 per cent of $167?
4. 11 per cent of $321?
5. 4 per cent of $328?
6. 5 per cent of $83?
7. 8 per cent of $257?

F. When the given per cent can be reduced to a more convenient vulgar fraction, by reducing it to its lowest terms, the following form is preferable.

1. What is $66\frac{2}{3}$ per cent of \$714?

SOLUTION. Since $66\frac{2}{3}$ per cent $= \frac{2}{3}$, $66\frac{2}{3}$ per cent of \$714 $= \frac{2}{3}$ of \$714 $=$ \$476.

What is—

2. 25 per cent of \$84?
3. $12\frac{1}{2}$ per cent of \$5.76?
4. $33\frac{1}{3}$ per cent of \$4.74?
5. 20 per cent of \$3.25?
6. $14\frac{2}{7}$ per cent of \$49?
7. $11\frac{1}{9}$ per cent of \$18.54?
8. 40 per cent of \$125?
9. 30 per cent of \$670?

G. 1. I bought a sleigh for \$16.20, and paid a sum equal to 8 per cent of the cost for having it repaired. How much did I pay for having it repaired?

2. George received 9 per cent of \$144, and William received 6 per cent of \$216. Which received the most?

3. A trader bought a lot of goods for \$36, and sold them so as to gain 10 per cent of the cost. What was his gain, and for how much did he sell them?

4. I bought goods for \$300, and sold them so as to gain 15 per cent. What was my gain?

5. I gave \$28.40 for a lot of goods, but lost $12\frac{1}{2}$ per cent of their cost. How many dollars did I lose, and for how many dollars did I sell them?

6. Mr. Brown bought a horse for \$150, and sold him at an advance, or gain, of $16\frac{2}{3}$ per cent. What was his gain?

7. I bought a carriage for \$175, and, after paying 12 per cent of the cost for repairing it, I sold it for \$225. Did I gain or lose, and how much?

H. 1. For how much must I sell goods costing \$3.50 to gain 4 per cent?

ABBREVIATED SOLUTION. I must sell them for \$3.50 + *4 per cent of* \$3.50 $=$ \$3.50 + \$.14 $=$ \$3.64.

2. For how much must I sell goods costing $7.25 to lose 12 per cent?

ABBREVIATED SOLUTION. I must sell them for $7.25 — 12 per cent of $7.25 = $7.25 — $.87 = $6.38.

For how much must I sell articles—

3. Costing $72 to gain 7 per cent?
4. Costing $54 to lose 3 per cent?
5. Costing $7.32 to gain 10 per cent?
6. Costing $5.94 to lose 10 per cent?
7. Costing $6 to gain 18 per cent?
8. Costing $25 to lose 20 per cent?
9. Costing $4.20 to lose $16\frac{2}{3}$ per cent?
10. Costing $5.40 to gain $66\frac{2}{3}$ per cent?

I. 1. What per cent shall I gain by selling goods costing $6 for $7?

SOLUTION. Since the goods cost $6 and were sold for $7 I shall gain $1, which, since the goods cost $6, is equal to $\frac{1}{6}$ of the cost, or to $16\frac{2}{3}$ per cent of the cost.

What per cent shall I gain or lose by selling goods costing—

2. $8 for $9?
3. $12 for $16?
4. $16 for $22?
5. $10 for $11?
6. $11 for $10?
7. $2 for $2.50?
8. $2.50 for $2?
9. $5 for $7.50?
10. 25 cents for $37\frac{1}{2}$ cents?
11. 50 cents for $62\frac{1}{2}$ cents?

J. 1. What is the cost of goods on which I gain 12 cents by gaining 15 per cent?

PARTIAL SOLUTION. 15 per cent or $\frac{15}{100} = \frac{3}{20}$; hence,

12 cents equals $\frac{3}{20}$ of the cost. If 12 cents $= \frac{3}{20}$ of the cost, $\frac{1}{20}$ of the cost must equal, &c.

What is the cost of goods on which I—

2. Gain \$15 by gaining $12\frac{1}{2}$ per cent?
3. Gain \$2.50 by gaining 20 per cent?
4. Gain \$1.25 by gaining 16 per cent?
5. Lose \$12 by losing 16 per cent?
6. Lose \$48 by losing 75 per cent?
7. Lose \$3.17 by losing $33\frac{1}{3}$ per cent?

K. 1. What is the cost of goods on which I gain 16 per cent by selling for \$1.45?

SOLUTION. Since I gain 16 per cent or $\frac{4}{25}$ of the cost, I must sell for $\frac{25}{25} + \frac{4}{25} = \frac{29}{25}$ of the cost, and as I sell for \$1.45 it follows that \$1.45 is $\frac{29}{25}$ of the cost. Hence, $\frac{1}{25}$ of the cost must be $\frac{1}{29}$ of \$1.45, which is \$.05, and $\frac{25}{25}$ of the cost must be 25 times \$.05 = \$1.25. Hence, the cost of goods on which, &c.

2. What is the cost of goods on which I lose 10 per cent by selling for \$3.60?

PARTIAL SOLUTION. Since I lose 10 per cent or $\frac{1}{10}$ of the cost, I must sell for $\frac{10}{10} - \frac{1}{10} = \frac{9}{10}$ of the cost, and as I sell for \$3.60 it follows that \$3.60 is $\frac{9}{10}$ of the cost. Hence, $\frac{1}{10}$ of the cost must be, &c.

What is the cost of goods on which I—

3. Gain $33\frac{1}{3}$ per cent by selling for \$28?
4. Gain 25 per cent by selling for \$12.50?
5. Lose 20 per cent by selling for \$56?
6. Lose $12\frac{1}{2}$ per cent by selling for \$8.40?
7. Gain 8 per cent by selling for \$1.62?
8. Lose 5 per cent by selling for \$1.33?
9. *Gain* 16 per cent by selling for \$87?

L. 1. What per cent of the selling price shall I gain if I gain 24 per cent of the cost?

SOLUTION. If I gain 24 per cent of the cost I shall sell for 124 per cent of the cost, and the gain, 24 per cent of the cost, must equal $\frac{24}{124} = \frac{6}{31} = 19\frac{1}{31}$, per cent of the selling price. Hence, I shall gain $19\frac{1}{31}$ per cent of the selling price, if I gain 24 per cent of the cost.

What per cent of the selling price shall I—

2. Gain if I gain 20 per cent of the cost?
3. Gain if I gain 35 per cent of the cost?
4. Gain if I gain 10 per cent of the cost?
5. Lose if I lose 10 per cent of the cost?
6. Lose if I lose 50 per cent of the cost?
7. Lose if I lose 30 per cent of the cost?
8. Gain if I gain $14\frac{2}{7}$ per cent of the cost?
9. Gain if I gain $11\frac{1}{9}$ per cent of the cost?

10. I lost 25 per cent of the cost of a horse by selling him for $120. What per cent of his cost did I receive? How many dollars did he cost me? How many dollars did I lose?

11. I gained $16\frac{2}{3}$ per cent of the cost of a horse by selling him for $140. What was his cost, and how many dollars did I gain?

12. By selling some cloth at 24 cents per yard, I should gain 15 per cent less than I should by selling it at 27 cents per yard. What was its cost?

13. By selling cloth at $12\frac{1}{2}$ cents per yard, I gain 25 per cent. For how much should I sell it to gain 50 per cent?

14. By selling a lot of goods for $84, I shall gain 8 per cent more than I should lose by selling them for $72. What was their cost?

SECTION XXXII.

A. The money which men charge for their services in buying or selling goods for others, is called COMMISSION, and is usually a certain per cent of the cost of the goods bought, and of the money received for those sold.

1. Mr. Clarke sold a lot of goods for Mr. Davis for $500, at a commission of 3 per cent. What did his commission amount to, and how much money would be left for Mr. Davis?

SUGGESTION. There would be left for Mr. Davis $500 minus Mr. Clarke's commission of 3 per cent of $500.

2. I sold a lot of goods for $250, at a commission of 4 per cent. What did my commission amount to, and what would be left for the owner of the goods?

3. A commission merchant sold 85 barrels of flour, at $8 per barrel, receiving a commission of 2 per cent. What was his commission, and what should he pay the owner of the goods?

4. I bought $360 worth of cloth for Mr. Arnold, charging him a commission of 2 per cent. What was my commission, and what ought Mr. Arnold to pay me for the cloth and my commission?

SUGGESTION. Mr. Arnold ought to pay me $360 plus my commission of 2 per cent. of $360.

5. George bought a jack-knife for James for 75 cents, charging a commission of 8 per cent. How much ought *James to pay for* the knife and George's commission?

6. Mr. Greene bought a lot of shoes for Mr. Gardner, for which he paid \$120, and charged $3\frac{1}{2}$ per cent commission. What ought Mr. Gardner to pay for the shoes and Mr. Greene's commission?

7. What is the cost of goods, if \$93 is sufficient to pay the cost and a commission of $3\frac{1}{3}$ per cent?

SOLUTION. Since \$93 includes the cost and the commission of $3\frac{1}{3}$ per cent of the cost, it must equal $103\frac{1}{3}$ per cent $= \frac{3}{3}\frac{1}{0}$ of the cost. The cost must equal $\frac{3}{3}\frac{0}{1}$ of \$93, $=$ \$90, and the commission must equal $\frac{1}{3}\frac{}{1}$ of \$93 $=$ \$3.

PROOF. $3\frac{1}{3}$ per cent of \$90 $=$ \$3, and the sum of \$90 + \$3 $=$ \$93 $=$ the sum given as the cost and commission.

8. What is the cost of goods, if \$78 will just pay the cost and commission of 4 per cent?

9. What is the cost of goods, if \$9.27 will pay the cost and commission of 3 per cent?

10. What is the cost of goods, if \$14.70 will pay the cost and commission of 5 per cent?

11. For what were goods sold at a commission of 6 per cent, when \$14.10 was returned to the owner?

SOLUTION. Since \$14.10 $=$ the difference between the selling price of the goods and the commission of 6 per cent or $\frac{5}{5}\frac{}{0}$ of the cost, it must equal $\frac{4}{5}\frac{7}{0}$ of the sum for which they were sold. Hence, they were sold for $\frac{5}{4}\frac{0}{7}$ of \$14.10 $=$ \$15.

12. For what must goods be sold at a commission of 5 per cent, in order that \$133 may be returned to the owner?

13. For what must goods be sold at a commission of 2 per cent, in order that \$54.30 may be returned to the owner?

14. For what must goods be sold at a commission of $2\frac{1}{2}$ per cent, in order that \$3.51 may be returned to the owner?

15. I bought cloth at 25 cents per yard, and sent it to a commission merchant, who sold it at an advance of 40 per cent on its cost, and charged me a commission of 10 per cent on the sale. For what did he sell the cloth per yard? What was his commission? What was my net gain on the cloth?

16. I bought a lot of goods for $168, and sent them to a commission merchant, who sold them at an advance of 25 per cent, charging a commission of 3 per cent on the sale. What was his commission, and what was my net gain?

17. I bought a lot of goods for $100. A commission merchant sold them for me at 25 per cent less than cost, charging a commission of 4 per cent. How much was his commission? What was my total loss on the goods?

SECTION XXXIII.

A. 1. If I should have the use of another man's horse for a day, or a week, I ought to pay for it; or if I should occupy a house or a store belonging to another, I ought to pay rent for the use of it. In like manner, if I should borrow a sum of money, I ought to pay for the use of it.

2. Money thus paid for the use of money, is called INTEREST.

3. The money lent or used is called the PRINCIPAL, and the principal and interest together, form the AMOUNT.

ILLUSTRATIONS. If I should pay $3 for the privilege of using $100 for six months, the $3 would be the *interest* of the $100 for 6 months; the $100 would be the *principal*, and $100 + $3, or *$103, would be the amount.*

4. The interest is usually a certain per cent of the principal for each year it is used. This per cent is called the RATE PER CENT, or simply the RATE.

ILLUSTRATION. If a man is to pay a sum equal to $\frac{6}{100}$ of the principal for each year he uses it, the rate is 6 per cent.

5. In computing interest, a month is reckoned at 30 days.

B. **1.** What is the interest of \$8 for 2 years 9 mo., at 4 per cent?

SOLUTION. At 4 per cent per year, the interest for 2 yr. 9 mo., or $2\frac{3}{4}$ years, must be $2\frac{3}{4}$ times 4 per cent, or 11 per cent of the principal. 11 per cent. of \$8 = 11 times 8 cents = 88 cents, or \$.88 = the answer.

What is the interest—

2. Of \$7 for 2 yr., at 6 per cent?
3. Of \$9 for 3 yr., at 5 per cent?
4. Of \$18 for 6 mo., at 6 per cent?
5. Of \$243 for 4 mo., at 6 per cent?
6. Of \$43.21 for 1 yr. 10 mo. at 6 per cent?
7. Of \$52.30 for 2 yr. 6 mo., at 4 per cent?
8. Of \$132 for 1 yr., at 7 per cent?
9. Of \$937 for 8 mo., at 6 per cent?
10. Of \$42.73 for 2 yr., at $4\frac{1}{2}$ per cent?
11. Of \$23.17 for 1 mo., at 6 per cent?
12. Of \$24.36 for 9 mo., at 8 per cent?
13. Of \$53.27 for 1 yr. 4 mo., at 6 per cent?

C. Interest is more frequently reckoned at 6 per cent per year, than at any other rate. Hence, in all the following examples and explanations, interest should be reckoned at 6 per cent, unless otherwise stated.

2 months being $\frac{1}{6}$ of a year, interest for 2 months at 6

per cent must equal $\frac{1}{6}$ of 6 per cent, or 1 per cent of the principal.

At 6 per cent per year, what is the interest for 2 months of—

1. $37?	4. $657?	7. $85.75?
2. $58?	5. $938?	8. $123.79?
3. $49?	6. $8238?	9. $437.28?

D. At 6 per cent interest for 2 months being 1 per cent of the principal, interest for 100 times 2 months, or 200 months, or 16 years 8 months, must be 100 per cent of the principal, which is the principal itself.

From this we compute the following table:—

At 6 per cent per year, interest for—

200 mo., or 16 yr. 8 mo., or 6000 days = principal.

$\frac{1}{2}$ of 200 mo., or 100 mo., or 8 yr. 4 mo. = $\frac{1}{2}$ of prin.

$\frac{1}{3}$ of 200 mo., or $66\frac{2}{3}$ mo., or 5 yr. 6 mo. 20 da. = $\frac{1}{3}$ of prin.

$\frac{1}{4}$ of 200 mo., or 50 mo., or 4 yr. 2 mo. = $\frac{1}{4}$ of prin.

$\frac{1}{5}$ of 200 mo., or 40 mo., or 3 yr. 4 mo. = $\frac{1}{5}$ of prin.

$\frac{1}{6}$ of 200 mo., or $33\frac{1}{3}$ mo., or 2 yr. 9 mo. 10 da. = $\frac{1}{6}$ of prin.

$\frac{1}{8}$ of 200 mo., or 25 mo., or 2 yr. 1 mo. = $\frac{1}{8}$ of prin.

$\frac{1}{10}$ of 200 mo., or 20 mo., or 1 yr. 8 mo. = $\frac{1}{10}$ of prin.

$\frac{1}{12}$ of 200 mo., or $16\frac{2}{3}$ mo., or 1 yr. 4 mo. 20 da. = $\frac{1}{12}$ of prin.

$\frac{1}{15}$ of 200 mo., or $13\frac{1}{3}$ mo., or 1 yr. 1 mo. 10 da. = $\frac{1}{15}$ of prin.

$\frac{1}{16}$ of 200 mo., or $12\frac{1}{2}$ mo., or 1 yr. 15 da. = $\frac{1}{16}$ of prin.

1. What is the interest of $60 for each time mentioned in the table?

ANSWER. The interest of $60 for 200 mo., or 16 yr. 8 *mo.* = *$60; for 100* mo., or 8 yr. 4 mo. = $\frac{1}{2}$ of $60 =

$30; for 66⅔ mo., or 5 yr. 6 mo. 20 da. = ⅓ of $60 = = $20, &c., &c.

2. What is the interest of $36 for each time mentioned in the table? Of $48.72?

E. What is the interest of—

1. $40 for 100 mo.?
2. $48 for 12½ mo.?
3. $66 for 16⅔ mo.?
4. $24.36 for 66⅔ mo.?
5. $16.98 for 25 mo.?
6. $84.60 for 20 mo.?
7. $42 for 50 mo.?
8. $37 for 40 mo.?
9. $54.72 for 33⅓ mo.?
10. $75.15 for 13⅓ mo.?
11. $64 for 4 yr. 2 mo.?
12. $24.60 for 5 yr. 6 mo. 20 da.?
13. $16.64 for 1 yr. 15 da.?
14. $25.75 for 3 yr. 4 mo.?
15. $44.36 for 1 yr. 8 mo.?
16. $16.24 for 8 yr. 4 mo.?
17. $44 for 2 yr. 1 mo.?
18. $60.45 for 1 yr. 1 mo. 10 da.?
19. $43.78 for 16 yr. 8 mo.?
20. $75 for 2 yr. 9 mo. 10 da.?

F. At 6 per cent, the interest for 20 mo., or 1 yr. 8 mo., or 600 days, being $\frac{1}{10}$ of the principal, may be found by removing the decimal point 1 place to the left, and is as many dimes as there are dollars in the principal.

Hence, at 6 per cent per year interest for—

½ of 20 mo., or 10 mo., or 300 da. = ½ of $\frac{1}{10}$ of principal.
⅓ of 20 mo., or 6 mo. 20 da., or 200 da. = ⅓ of $\frac{1}{10}$ of prin.
¼ of 20 mo., or 5 mo., or 150 da. = ¼ of $\frac{1}{10}$ of prin.
⅕ of 20 mo., or 4 mo., or 120 da. = ⅕ of $\frac{1}{10}$ of prin.
⅙ of 20 mo., or 3 mo. 10 da., or 100 da. = ⅙ of $\frac{1}{10}$ of prin.
⅛ of 20 mo., or 2 mo. 15 da., or 75 da. = ⅛ of $\frac{1}{10}$ of prin.
$\frac{1}{12}$ of 20 mo., or 1 mo. 20 da., or 50 da. = $\frac{1}{12}$ of $\frac{1}{10}$ of prin.
$\frac{1}{15}$ of 20 mo., or 1 mo. 10 da., or 40 da. = $\frac{1}{15}$ of $\frac{1}{10}$ of prin.

1. What is the interest of $24 for each time mentioned in the table?

ANSWER. The interest of \$24 for 10 mo., or 300 da. $= \frac{1}{2}$ of \$2.40 = \$1.20; for $6\frac{2}{3}$ mo., or 6 mo. 20 da., or 200 da. $= \frac{1}{3}$ of \$2.40 = \$.80, &c.

2. What is the interest of \$120 for each time mentioned in the table? Of \$7.50? Of \$4.86?

What is the interest of—

3. \$72 for $2\frac{1}{2}$ mo.?
4. \$60 for $1\frac{1}{3}$ mo.?
5. \$486 for $1\frac{2}{3}$ mo.?
6. \$15 for 10 mo.?
7. \$2.40 for $6\frac{2}{3}$ mo.?
8. \$64.50 for 120 da.?
9. \$36.60 for 100 da.?
10. \$483.60 for 50 da.?
11. \$27 for 5 mo.?
12. \$7.50 for 75 da.?
13. \$74.10 for 3 mo. 10 da.?
14. \$55 for 1 mo. 10 da.?
15. \$1.86 for 200 da.?
16. \$54.20 for 4 mo.?

G. At 6 per cent the interest for 2 months, or 60 days, being $\frac{1}{100}$ of the principal, it follows that the interest for—

$\frac{1}{2}$ of 2 mo., or 1 mo., or 30 da. $= \frac{1}{2}$ of $\frac{1}{100}$ of the principal.
$\frac{1}{3}$ of 2 mo., or 20 da. $= \frac{1}{3}$ of $\frac{1}{100}$ of the prin.
$\frac{1}{4}$ of 2 mo., or 15 da. $= \frac{1}{4}$ of $\frac{1}{100}$ of the prin.
$\frac{1}{5}$ of 2 mo., or 12 da. $= \frac{1}{5}$ of $\frac{1}{100}$ of the prin.
$\frac{1}{6}$ of 2 mo., or 10 da. $= \frac{1}{6}$ of $\frac{1}{100}$ of the prin.
$\frac{1}{10}$ of 2 mo., or 6 da. $= \frac{1}{10}$ of $\frac{1}{100}$ or $\frac{1}{1000}$ of the prin.
$\frac{1}{12}$ of 2 mo., or 5 da. $= \frac{1}{12}$ of $\frac{1}{100}$ of the prin.
$\frac{1}{2}$ of 6 da., or 3 da. $= \frac{1}{2}$ of $\frac{1}{1000}$ of the prin.
$\frac{1}{3}$ of 6 da., or 2 da. $= \frac{1}{3}$ of $\frac{1}{1000}$ of the prin.
$\frac{1}{6}$ of 6 da., or 1 da. $= \frac{1}{6}$ of $\frac{1}{1000}$ of the prin.

1. What is the interest of \$432 for each time mentioned in the table?

SOLUTION. The interest of \$432 for 2 mo., or 60 da., is \$4.32; for 1 mo., or 30 da., is $\frac{1}{2}$ of \$4.32, which is \$2.16, &c., &c., * * * for 6 days, is \$.432; for 3 days, is $\frac{1}{2}$ of *\$.432, which is* \$.216, &c., &c.

2. What is the interest of $360 for each time mentioned in the table? Of $60.30?

What is the interest of—

3. $42 for 20 da.?
4. $36.24 for 15 da.?
5. $48 for 10 da.?
6. $89 for 6 da.?
7. $174 for 3 da.?
8. $192 for 5 da.?
9. $43.50 for 12 da.?
10. $86.37 for 30 da.?
11. $228 for 1 da.?
12. $234 for 2 da.?

H. The foregoing principles furnish short and expeditious methods of computing interest for any time whatever.

1. What is the interest of $72.60 for 8 mo. 20 da.?

FIRST SOLUTION. 8 mo. 20 da. = 6 mo. 20 da. + 2 mo. The interest of $72.60 for 6 mo. 20 da. = $\frac{1}{3}$ of $7.26 = $2.42, and the interest for 2 mo. = $.726, which, added to $2.42 = $3.146 = Ans.

SECOND SOLUTION. 8 mo. 20 da. = 10 mo. — 1 mo. 10 da. The interest of $72.60 for 10 mo. = $\frac{1}{2}$ of $7.26 = $3.63, and the interest for 1 mo. 10 da. = $\frac{1}{15}$ of $7.26 = $.484, which, subtracted from $3.63 = $3.146 = Ans.

THIRD SOLUTION. 8 mo. 20 da. = 8 mo. + 20 da. The interest of $72.60 for 8 mo. or 4 times 2 mo. = 4 per cent of $72.60 = $2.904, and the interest for 20 da. = $\frac{1}{3}$ of $.726 = $.242, which, added to $2.904 = $3.146 = Ans.

The work can be written as follows:—

FIRST SOLUTION.	SECOND SOLUTION.
$72.60 = prin.	$72.60 = prin.
2.42 = int. 6 mo. 20 da.	3.63 = int. 10 mo.
.726 = int. 2 mo.	.484 = int. 1 mo. 10 da.
$3.146 = int. 8 mo. 20 da.	$3.146 = int. 8 mo. 20 da.

NOTE. The form for writing the third solution would be similar to the above forms.

What is the interest of—

2. $90 for 3 mo. 16 da.?
3. $128 for 22 mo. 15 da.?
4. $64 for 2 mo. 10 da.?
5. $32 for 5 mo. 15 da.?
6. $120.90 for 3 mo. 20 da.?
7. $88.24 for 5 mo. 6 da.?
8. $72.96 for 1 mo. 26 da.?
9. $500 for 9 mo. 24 da.?
10. $1000 for 3 mo. 29 da.?
11. $54.24 for 7 mo. 15 da.?
12. $150 for 35 mo. 10 da.?
13. $184 for 2 yr. 3 mo.?
14. $96 for 52 mo. 15 da.?
15. $186.60 for 7 mo.?
16. $28.16 for 2 yr. 6 mo.?
17. $384 for 19 mo. 27 da.?
18. $30.24 for 6 mo. 17 da.?
19. $450.36 for 3 mo. 15 da.?

I. Business men often use such methods as the following in connection with those already explained:—

At 6 per cent per year the interest of $2 for 1 month is 1 cent. Hence, at 6 per cent—

The interest of $2 is 1 cent per month.
The interest of $20 is 1 dime per month.
The interest of $200 is 1 dollar per month.

1. What is the interest of $2 for each of the following times?

2. 3 mo.?
3. 9 mo.?
4. 15 mo.?
5. 2 yr. 3 mo.?
6. 1 yr. 5 mo.?
7. 2 yr. 1½ mo.?
8. 15⅓ mo.?
9. 4 mo. 10 da.?
10. 2 yr. 7 mo.?

What is the interest for each of the above times of—

11. $1?
12. $6?
13. $8?
14. $20?
15. $5?
16. $10?
17. $200?
18. $50?
19. $500?
20. $14?
21. $80?
22. $800?

J. At 6 per cent per year—

The interest of $6 is 1 mill per day.
The interest of $60 is 1 cent per day.
The interest of $600 is 1 dime per day.
The interest of $6000 is 1 dollar per day.

What is the interest of $6 for each of the following times?

1. 3 da.?	4. 1 mo. 3 da., or 33 da.?	7. 3 mo. 6 da.?
2. 7 da.?	5. 1 mo. 17 da.?	8. 5 mo. 12 da.?
3. 19 da.?	6. 2 mo. 25 da.?	9. 4 mo. 9 da.?

What is the interest for each of the above times of—

10. $60?	14. $600?	18. $6000?
11. $30?	15. $300?	19. $1000?
12. $20?	16. $150?	20. $1500?
13. $120?	17. $1800?	21. $500?

K. 1. What principal will gain $3.60 in 1 yr. 9 mo. 10 da., at 9 per cent?

SOLUTION. At 9 per cent per year, interest for 1 yr. 9 mo. 10 da. or $1\frac{7}{9}$ yr. = 16 per cent, or $\frac{4}{25}$ of the principal. Hence, $3.60 = $\frac{4}{25}$ of the required principal, and the required principal = $\frac{25}{4}$ of $3.60 = $22.50.

What principal will gain—

2. $13 in 1 yr. 3 mo., at 4 per cent?
3. $24 in 1 yr. 6 mo., at 5 per cent?
4. $49 in 2 yr. 4 mo., at 3 per cent?
5. $1.80 in 2 yr., at $7\frac{1}{2}$ per cent?
6. $81 in 9 mo., at 6 per cent?
7. $72 in 2 yr. 4 mo. 24 da., at 5 per cent?
8. $15 in 16 mo. 20 da., at 6 per cent?
9. $71 in 25 mo., at 6 per cent?
10. $5 in $3\frac{1}{3}$ mo., at 6 per cent?

L. 1. What principal will amount to $54 in 1 yr. 4 mo., at 6 per cent?

ABBREVIATED SOLUTION. At 6 per cent per year, interest for 1 yr. 4 mo. = 8 per cent = $\frac{2}{25}$ of the principal. Hence, the amount must equal $1\frac{2}{25}$ times the principal = $\frac{27}{25}$ of the principal, and the principal must equal $\frac{25}{27}$ of the amount. Hence, $\frac{25}{27}$ of $54 = $50 = the principal required.

What principal will amount to—

2. $63 in 2 yr. 9 mo. 10 da., at 6 per cent?
3. $88 in 1 yr. 8 mo., at 6 per cent?
4. $8.10 in 2 yr. 6 mo., at 5 per cent?
5. $168 in 3 yr., at 4 per cent?
6. $57.57 in 4 yr., at 3½ per cent?
7. $144 in 2 yr. 1 mo , at 6 per cent?
8. $72.36 in 3 yr. 4 mo., at 6 per cent?
9. $2.21 in 1 yr. 6 mo., at 7 per cent?
10. $50 in 4 yr., at 6¼ per cent?
11. $9.32 in 5 yr. 6 mo. 20 da., at 6 per cent?

SECTION XXXIV.

A. 1. A and B traded in company. A put in $60, and B put in $40. They gained $75. What was each man's just share?

PARTIAL SOLUTION. Both together put in $100, of which A's $60 was $\frac{3}{5}$, and B's $40 was $\frac{2}{5}$. Hence, A should have $\frac{3}{5}$ of the gain, and B should have $\frac{2}{5}$ of it.

A and B trade in company. How many dollars will each man gain or lose, if, when A puts in—

2. $18 and B $54, they gain $36?
3. $15 and B $25, they gain $72?
4. $18 and B $36, they lose $15?
5. $4½ and B $6, they gain $4.20?
6. $6¼ and B $2 1/12, they lose $2.40?
7. $⅔ and B $½, they gain $1.05?
8. $ 9/10 and B $⅗, they lose $1.00?
9. $4½ and B $10½, they gain $5.00?
10. *$3 as often* as B puts in $7, they gain $42?

PARTIAL SOLUTION. For every \$10 put in by both, A puts in \$3 and B puts in \$7. Hence, A puts in $\frac{3}{10}$ of the stock and should have $\frac{3}{10}$ of the gain, and B puts in $\frac{7}{10}$ of the stock and should have $\frac{7}{10}$ of the gain.

A and B trade in company. How many dollars will each gain or lose, if, when A puts in—

11. \$4 as often as B puts in \$5, they gain \$40?
12. \$$\frac{1}{2}$ as often as B puts in \$$\frac{1}{8}$, they gain \$42?
13. \$2$\frac{1}{3}$ as often as B puts in \$1$\frac{3}{4}$, they lose \$100?
14. $\frac{2}{3}$ as much as B, they gain \$17?

PARTIAL SOLUTION. Since A puts in $\frac{2}{3}$ as much as B, both must put in 1$\frac{2}{3}$, or $\frac{5}{3}$ times as much as B. Hence, A must put in $\frac{2}{5}$ of the whole stock, and B must put in $\frac{3}{5}$ of it.

A and B trade in company. How many dollars will each gain or lose, if, when A puts in—

15. $\frac{3}{5}$ as much as B, they lose \$50?
16. $\frac{4}{7}$ as much as B, they gain \$30?
17. 3 times as much as B, they lose \$1000?
18. 2$\frac{1}{2}$ times as much as B, they gain \$13?
19. $\frac{4}{9}$ of the stock B, gains \$5 more than A?

PARTIAL SOLUTION. Since A puts in $\frac{4}{9}$ of the stock, B must put in $\frac{5}{9}$ of it, or $\frac{1}{9}$ of it more than A. Hence, he must have $\frac{1}{9}$ of the gain more than A, or \$5 = $\frac{1}{9}$ of the gain of both.

A and B trade in company. How many dollars will each gain, if, when A puts in—

20. $\frac{2}{7}$ of the stock, B gains \$12 more than A?
21. $\frac{7}{11}$ of the stock, B gains \$25 less than A?
22. $\frac{3}{8}$ of the stock, B loses \$13 more than A?
23. $\frac{7}{12}$ of the stock, B receives \$6 more than $\frac{1}{2}$ the gain?
24. $\frac{2}{3}$ of the stock, B has \$4 more than $\frac{1}{8}$ of the gain?

B. A, B, and C trade in company. How many dollars will each gain or lose, if, when A puts in—

1. \$3, B \$4, and C \$2, they gain \$28?
2. \$1, B \$2, and C \$3, they gain \$75?
3. \$6, B \$3, and C \$15, they gain \$80?
4. \$$\frac{1}{2}$, B \$$\frac{1}{3}$, and C \$$\frac{1}{4}$, they gain \$1.17?
5. \$2$\frac{1}{3}$, B \$4$\frac{1}{2}$, and C \$1$\frac{1}{6}$, they lose \$1.44?
6. $\frac{1}{2}$ as much as B, or $\frac{2}{3}$ as much as C, they gain \$72?

PARTIAL SOLUTION. Since A puts in $\frac{1}{2}$ as much as B, or $\frac{2}{3}$ as much as A, B puts in twice as much as A, and C puts in $\frac{3}{2}$ or $1\frac{1}{2}$ times as much as A. Hence, all put in $1 + 2 + 1\frac{1}{2} = 4\frac{1}{2}$ times as much as A, and A should have $\frac{1}{4\frac{1}{2}} = \frac{2}{9}$ of the gain, B should have $\frac{2}{4\frac{1}{2}} = \frac{4}{9}$ of the gain, and C should have $\frac{1\frac{1}{2}}{4\frac{1}{2}} = \frac{3}{9}$ of the gain.

A, B, and C trade in company. How many dollars will each gain or lose, if, when A puts in—

7. $\frac{4}{3}$ as much as B, or $\frac{3}{4}$ as much as C, they gain \$74?
8. $\frac{6}{5}$ as much as B, or $\frac{3}{2}$ as much as C, they lose \$45?
9. As much as B, or twice as much as C, they lose \$2?
10. As much as B, or $1\frac{1}{3}$ times as much as C, they gain \$7.70?
11. \$4, B \$2, and C \$6, A gains \$1 more than B?
12. \$100, B \$150, and C \$250, C gains \$96 more than A?
13. \$10, B \$9, and C \$6, A gains \$10 less than B and C together?
14. $\frac{2}{7}$ of the stock, and B puts in twice as much as C, they gain \$420?

PARTIAL SOLUTION. Since A puts in $\frac{2}{7}$ of the stock, B and C together must put in the remaining $\frac{5}{7}$ of it. Again, since B puts in twice as much as C, both must put in 3 *times as much* as C, or C must put in $\frac{1}{3}$ and B $\frac{2}{3}$ of what

both put in. It follows, then, that B must put in $\frac{2}{3}$ of $\frac{5}{7} = \frac{10}{21}$ of the whole stock, and that C must put in $\frac{1}{3}$ of $\frac{5}{7} = \frac{5}{21}$ of the whole stock. Hence, A will have $\frac{2}{7}$, B $\frac{10}{21}$, and C $\frac{5}{21}$ of the gain.

A, B, and C trade in company. How many dollars will each gain or lose, if, when A puts in—

15. $\frac{1}{3}$ of the stock, and B puts in 3 times as much as C, they gain \$720?

16. $\frac{8}{10}$ of the stock, and B puts in $\frac{1}{2}$ as much as C, they gain \$540?

17. $\frac{2}{11}$ of the stock, and B puts in $\frac{4}{5}$ as much as C, they lose \$143?

18. $\frac{1}{8}$ of the stock, and B puts in $1\frac{1}{3}$ times as much as C, they lose \$136?

19. $\frac{1}{6}$ of the stock, and B puts in $\frac{4}{3}$ as much as C, they gain \$72?

20. $\frac{3}{8}$ of the stock, and B puts in $\frac{2}{3}$ of the remainder, C gains \$48?

21. $\frac{1}{12}$ of the stock, and B puts in $\frac{5}{6}$ as much as C, B gains \$30?

22. $\frac{5}{16}$ of the stock, and C puts in $1\frac{1}{5}$ times as much as A, B gains \$100?

23. $\frac{1}{3}$ as much as B and C together, and B puts in $\frac{1}{2}$ as much as C, they gain \$100?

24. $\frac{4}{5}$ as much as B and C together, and B puts in 1 times as much as C, they gain \$189?

C. The foregoing problems and solutions are based on the supposition that the stock of all the partners has been in trade for the same length of time. If, however, the stocks are invested for unequal times, the gain or loss should be shared among the partners in proportion to what the use or

interest of their several stocks is worth, for the time they have been invested.

1. A, B, and C traded in company. A paid in \$5 for 4 mo., B \$3 for 5 mo., and C \$4 for $4\frac{1}{2}$ mo. They gained \$159. How many dollars was each person's just share?

PARTIAL SOLUTION. The use of A's \$5 for 4 mo. was worth as much as the use of \$1 for 5 times 4 mo. or for 20 mo. The use of B's \$3 for 5 mo. was worth as much as the use of \$1 for 3 times 5 mo. or for 15 mo. The use of C's \$4 for $4\frac{1}{2}$ mo. was worth as much as the use of \$1 for 4 times $4\frac{1}{2}$ mo. or for 18 mo. Hence, the use of the entire stock for the time it was in trade was worth as much as the use of \$1 for 20 mo. + 15 mo. + 18 mo., or for 53 mo., and A should have $\frac{20}{53}$, B $\frac{15}{53}$, and C $\frac{18}{53}$ of the gain.

2. A, B, and C traded in company. A put in \$9 for 6 mo., B \$8 for 7 mo., and C \$10 for 9 mo. They gained \$100. What was each man's share?

3. A, B, and C traded in company. A put in \$7 for 9 mo., B \$8 for 8 mo., and C \$10 for $9\frac{4}{5}$ mo. They gained \$45. What was each man's share?

4. X, Y, and Z hired a pasture for \$53. A put in 20 sheep for 9 weeks, Y put in 15 sheep for 10 weeks, and Z put in 25 sheep for 8 weeks. How many dollars ought each to pay?

5. James, George, and Edward together did a piece of work for which they received \$75. James worked 10 hours per day for 4 days, and 9 hours per day for 6 days. George worked 8 hours per day for 2 days, 11 hours per day for 3 days, and 9 hours per day for 5 days. Edward worked $9\frac{1}{3}$ hours per day for 6 days, and $9\frac{1}{4}$ hours per day for 4 days. How many dollars did each receive?

6. Joseph and John traded in company. Joseph put in

$\frac{3}{2}$ as much money as John, but kept it in only $\frac{3}{4}$ as long. If they gained \$51, how many dollars did each gain?

PARTIAL SOLUTION. The use of $\frac{3}{2}$ of John's money for $\frac{3}{4}$ as long a time as John's was invested, must be worth $\frac{3}{2}$ of $\frac{3}{4} = \frac{9}{8}$ as much as the use of John's money for the time it was in trade. Hence, the use of the entire stock of both was worth $1\frac{9}{8} = \frac{17}{8}$ as much as the use of John's stock; and John will have $\frac{8}{17}$ and Joseph will have $\frac{9}{17}$ of the gain.

7. Francis and Thomas traded in company. Francis put in only $\frac{5}{6}$ as much money as Thomas, but he kept it in trade $\frac{7}{5}$ as long. If they gained \$91, what was each partner's share of the gain?

8. A and B traded in company. A put in $2\frac{2}{3}$ times as much money as B, but kept it in trade only $\frac{3}{8}$ as long. They lost \$500. What was each man's loss?

SECTION XXXV.

NOTE. The solutions of this section are designed to illustrate a few of the many methods which may be used to abbreviate numerical work. The careful student can discover abbreviated processes in nearly all classes of problems.

A. 1. How many acres are there in a road 37 miles long and 4 rods wide?

SOLUTION. A road 1 mile, or 320 rods, long and 1 rod wide contains 320 sq. rd. = 2 acres. Hence, for every mile in length, a road 1 rod wide will contain 2 acres, and a road 4 rods wide will contain 4 times 2 acres = 8 acres. It follows, then, that a road 37 miles long and 4 rods wide must contain 37 times 8 acres = 296 acres.

How many acres are there in a road—

2. 24 miles long and 3 rods wide?
3. 53 miles long and $3\frac{1}{2}$ rods wide?
4. $31\frac{1}{2}$ miles long and 4 rods wide?
5. 63 miles long and $3\frac{1}{4}$ rods wide?
6. 75 miles long and $2\frac{2}{3}$ rods wide?
7. 96 miles long and $3\frac{3}{4}$ rods wide?

B. 1. How many acres are there in a field 25 rods long and 20 rods wide?

SOLUTION. A field 1 rod long and 20 rods wide would contain 20 sq. rd. $= \frac{1}{8}$ of an acre. Hence, a field 25 rods long and 20 rods wide will contain $\frac{25}{8}$ of an acre $= 3\frac{1}{8}$ acres.

How many acres are there in a field—

2. 40 rods long and 35 rods wide?
3. 35 rods long and 32 rods wide?
4. 80 rods long and 63 rods wide?
5. 43 rods long and 16 rods wide?
6. 160 rods long and 153 rods wide?
7. 75 rods long and 64 rods wide?
8. 92 rods long and 78 rods wide?

SUGGESTION. Since 78 = 2 less than 80, a field 92 rods long and 78 rods wide will contain twice 92 sq. rd. less than a field 92 rods long and 80 rods wide.

How many acres are there in a field—

9. 159 rods long and 94 rods wide?
10. 38 rods long and 21 rods wide?
11. 82 rods long and 63 rods wide?
12. 79 rods long and 73 rods wide?
13. 53 rods long and 319 rods wide?

C. 1. How many shingles will be required to cover a

surface 17 ft. long and 9 ft. wide, if each shingle covers a space 6 inches long and 4 inches wide?

SUGGESTION. Each shingle being $\frac{1}{2}$ ft. long and $\frac{1}{3}$ ft. wide will cover $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$ of a sq. ft. Hence, there will be 6 shingles for every sq. ft. of surface.

If one shingle covers a space 6 inches long and 4 inches wide, how many shingles will be required to cover a surface—

2. 23 ft. long and 20 ft. wide?
3. 24 ft. long and 21 ft. wide?
4. 19 ft. long and 13 ft. wide?

5. What would be the answers to the last three questions, if each shingle should cover a space 6 inches long by 3 wide? 8 inches long by 6 inches wide?

D. 1. How many square yards are there in a surface 21 ft. long and 17 ft. wide?

PARTIAL SOLUTION. A surface 21 ft. long and 1 ft. wide would contain 21 sq. ft. $= 2\frac{1}{3}$ sq. yd. Hence, a surface 21 ft. long and 17 ft. wide must contain 17 times $2\frac{1}{3}$ sq. yd.

How many square yards are there in a surface—

2. 18 ft. long and 13 ft. wide?
3. 54 ft. long and 43 ft. wide?
4. 39 ft. long and 30 ft. wide?
5. 33 ft. long and 8 ft. wide?
6. 63 ft. long and 47 ft. wide?

E. 1. How many cords of wood are there in a pile 48 ft. long, 4 ft. wide, and 6 ft. high?

FIRST SOLUTION. A pile of wood 4 ft. wide, 6 ft. high, and 1 ft. long, would contain 24 cu. ft. $= \frac{3}{16}$ of a cord.

Hence, a pile 48 ft. long contains 48 times $\frac{3}{16}$ of a cord, = 9 cords.

SECOND SOLUTION. A pile of wood 48 ft. long, 1 ft. wide, and 1 ft. high contains 48 cubic feet = $\frac{3}{8}$ of a cord. Hence, a pile 48 ft. long 4 ft. wide and 1 ft. high contains 4 times $\frac{3}{8}$ of a cord = $1\frac{1}{2}$ cords; and a pile 48 ft. long 4 ft. wide and 6 ft. high contains 6 times $1\frac{1}{2}$ cords = 9 cords.

How many cords of wood are there in a pile—

2. 67 ft. long, 4 ft. wide, and 8 ft. high?
3. 32 ft. long, 4 ft. wide, and 4 ft. high?
4. 128 ft. long, 4 ft. wide, and 3 ft. high?
5. 64 ft. long, 16 ft. wide, and 4 ft. high?
6. 96 ft. long, 4 ft. wide, and 7 ft. high?

F. 1. What will 32 books cost at 48 cents each?

SOLUTION. Since 48 cents = 2 cents less than $\frac{1}{2}$ of a dollar, 32 books at 48 cents each will cost 32 times 2 cents less than 32 half dollars = \$16 — \$.64 = \$15.36.

What will—

2. 23 caps cost, at 99 cents each?
3. 59 books cost, at 98 cents each?
4. 84 knives cost, at 24 cents each?
5. 12 razors cost, at 73 cents each?
6. 75 shawls cost, at \$1.36 each?

SOLUTION. 75 shawls at 1 cent each will cost 75 cents = $\frac{3}{4}$ of a dollar. Hence, at 136 cents each they will cost 136 times $\frac{3}{4}$ of a dollar = \$102.

How much will—

7. 50 articles cost, at \$4.28 each?
8. 25 articles cost, at \$2.72 each?
9. $33\frac{1}{3}$ articles cost, at \$9.53 each?
10. $12\frac{1}{2}$ articles cost, at \$5.44 each?

SECTION XXXVI.

NOTE. Exercises like the following will be found very interesting to a class, and very practical.

A. 1. What are the coins of the United States? See Section IX., A.

2. If you should have coins of each kind coined in the United States, and should wish to use the smallest possible number of coins in the payment, with what coins would you pay a debt of 17 cents?

ANSWER. With a dime, a half-dime, and 2 cents; or with a dime, 2 three-cent pieces, and a cent.

The same suppositions continuing, with what coins would you pay a debt of—

3. $.06?	8. $.75?	13. $12.50?
4. $.31?	9. $ 50?	14. $3.37?
5. $.83?	10. $.95?	15. $4.00?
6. $.67?	11. $3.00?	16. $1.62?
7. $.34?	12. $7.60?	17. $2.89?

18. Find which of the above debts can be paid with 4 coins, and name the coins for each.

19. Find which of the above debts can be paid with 5 coins, and name the coins for each.

20. Find which of the above debts can be paid with 6 coins, and name the coins for each.

B. Bank-bills are often used as a substitute for "Specie," *i. e.*, for gold and silver coins. These bills most commonly represent some one of the following values; viz.—$1, $2, $3, $5, $10, $20, $50, $100, $500, $1000.

If you should have coins of each denomination coined in the United States, and bank-bills of each of the above denominations, and should wish to use the smallest possible number of bills and coins in the payment of each debt how will you pay a debt of—

1. $4.25?	4. $13.21?	7. $7.65?
2. $8.37?	5. $163.87?	8. $99.99?
3. $4.50?	6. $68.13?	9. $19.84?

C. It often happens that a person does not have such coins or bank-bills as will enable him to pay the exact sum which he owes. In such cases it is customary for him to give such coins or bills as will make some convenient value greater than the amount to be paid, and receive in return a sum equal to the excess of what he has paid over what he owes. This is called "Making Change."

1. If each of two persons have coins of each kind coined in the United States, and wish to use the smallest possible number of coins in the payment of each debt, how can one pay the other $.19?

ANSWER. By giving a quarter-dollar and receiving back either 2 three-cent pieces, or a half-dime and a cent; or by giving 2 dimes and receiving back one cent.

The same suppositions continuing, how can one person pay the other a debt of—

2. $.38?	5. $.82?	8. $1.13?
3. $.63?	6. $.72?	9. $2.42?
4. $.29?	7. $.59?	10. $2.59?

D. If Otis has 10 half-dimes, and Asa has 15 three-cent pieces, how must change be made when Asa pays Otis—

1. 1 ct.?	4. 18 cts.?	7. 11 cts.?
2. 7 cts.?	5. 25 cts.?	8. 19 cts.?
3. 13 cts.?	6. 28 cts.?	9. 26 cts.?

10. The same suppositions continuing, how must change be made when Otis pays Asa each of the above sums?

E. Mr. Brooks has 2 quarter-eagles, 1 three-dollar bill, 4 silver dollars, and 9 three-cent pieces. Mr. Upham has 1 half-eagle, 3 two-dollar bills, 1 gold dollar, 7 half-dimes, and 2 cents. In what ways can change be so made that Mr. Brooks shall pay Mr. Upham a debt of—

1. $.20?	4. $.50?	7. $.47?
2. $.40?	5. $5.25?	8. $.89?
3. $4.50?	6. $.31?	9. $5.58?

F. Two persons have each a half-dollar, a quarter-dollar, a dime, a three-cent piece, and a cent. How can change be so made that one shall pay the other a debt of—

1. 8 cts.?	4. 18 cts.?	7. 87 cts.?
2. 16 cts.?	5. 28 cts.?	8. 69 cts.?
3. 17 cts.?	6. 61 cts.?	9. 56 cts.?

SECTION XXXVII.

MISCELLANEOUS PROBLEMS.

1. 7 times 8, plus 4, divided by 5, multiplied by 3, minus 4, minus 8, divided by 2, divided by 3, multiplied by 9, multiplied by 2 = how many times 8?

2. Multiply $\frac{1}{2}$ of 18 by $\frac{3}{4}$ of 8, add $\frac{1}{3}$ of 27, divide by $\frac{1}{4}$ of 28, add $\frac{1}{11}$ of 33, and square the number.

3. If $\frac{6}{7}$ of a yard of cloth worth 14 cents per yard, are given for $\frac{3}{5}$ of a pound of chocolate, how many pounds of coffee at 12 cents per pound should be given for $3\frac{1}{2}$ pounds of chocolate?

4. If I should expend the sum of \$9 + \$8 + \$5 + \$9 + \$4 + \$7 for flour at \$7 per barrel, and sell the flour at \$8 per barrel, then expend the proceeds for cloth at \$3 per yard, and sell the cloth for \$4 per yard, and then, after spending \$10, and losing \$6, should expend the remainder for tea at the rate of 3 pounds for \$2, how many pounds of tea should I buy?

5. Find the cost of $1\frac{3}{4}$ lb. of coffee at 16 cents per lb., $2\frac{1}{2}$ lb. of raisins at 8 cents per lb., $2\frac{2}{3}$ lb. of figs at 15 cents per lb., $4\frac{1}{3}$ lb. of sugar at 9 cents per lb., $1\frac{2}{5}$ lb. of tea at 40 cents per lb., and $\frac{3}{4}$ lb. of cotton at 32 cents per lb.

6. What must be the length of the side of a square field containing $\frac{1}{2}$ as many square rods as a field 9 rods long and 8 rods wide?

7. Arthur sold a certain number of apples at the rate of 2 for a cent, and Robert sold as many at the rate of 3 for a cent. Arthur received 12 cents more than Robert. How many apples did each sell?

8. A thief drew $\frac{1}{4}$ of the wine out of a certain cask, and, to escape detection, filled it with water. The next night he drew out $\frac{1}{4}$ of the contents of the cask, and again filled it with water. How many gills of wine will there now be in each gallon of the mixture?

9. $2\frac{1}{2}$ times a certain number added to $\frac{1}{3}$ of that number is $5\frac{1}{2}$ less than 3 times the number. What is the number?

10. A lady being asked her age, replied, "My father is 30 years older than my sister Sarah, and $8\frac{1}{3}$ times the difference between their ages is 5 times my father's age. Now, if you will tell how old my father and sister are, I will tell you how to find my age." A correct answer having been given, the lady said, "To 3 times my father's age, add 6 *times my* sister's age, and you will obtain a sum $\frac{1}{3}$ of which

is 9 years more than $4\frac{1}{2}$ times my age." What was the age of each?

11. If $\frac{1}{2}$ of any number whatever be added to $\frac{1}{3}$ of the same number, and the sum thus obtained be divided by $\frac{1}{6}$ of the same number, the quotient will always be 5. Why is this?

12. The sum of two numbers is 50, but if the smaller be subtracted from twice their sum the remainder is 77. What are the numbers?

13. A teacher wishing to obtain a black-board 15 ft. long and 6 ft. wide, bought boards for the purpose at $2\frac{1}{2}$ cents per square foot. He hired a carpenter to make it, paying him 75 cents for his work. He paid 11 cents per square yard to have it painted and varnished, and it cost 25 cents to have it brought to the school-room and put up. What was the whole cost?

14. My parlor and sitting-room are each 5 yards wide, but my parlor is 2 yards longer than my sitting-room. The floor of my sitting-room contains 30 square yards. What is the length of my parlor floor, and how many square yards does it contain?

15. David said to Harry, "If $\frac{1}{2}$ the sum of our ages be added to $\frac{1}{2}$ of your age, the same will equal $\frac{3}{4}$ of my age, and I am 12 years older than you are. What was the age of each of the boys?

16. Mr. Warren bought a cask of oil at \$1.20 per gallon, but $\frac{1}{5}$ of it leaked out. For how much per gallon must he sell the rest so as neither to gain nor lose?

17. Mr. Allen owes Mr. Mason 62 cents, and the only coins he has are 1 half-dollar, 1 quarter-dollar, 1 half-dime, and 2 three-cent pieces, while the only coins Mr. Mason has are 4 half-dollars, 5 dimes, and 2 cents. How can change be made so that the debt may be paid with these coins?

18. What number added to $\frac{2}{5}$ of itself equals 36 more than $\frac{1}{2}$ of the number?

19. A man sold 6 barrels of apples and 2 barrels of pears for $23, receiving twice as much per barrel for the pears as for the apples. How many dollars did he receive for each?

20. By selling cloth at $3.50 per yard, I lose $12\frac{1}{2}$ per cent of its cost. How many dollars should I lose on each yard by selling it at $3 per yard?

21. I sold $\frac{1}{2}$ of a lot of grain for what $\frac{2}{3}$ of it cost, thereby gaining $16. How much did the entire lot cost me?

22. A and B traded in company. A put in $360, and B put in $\frac{3}{5}$ of $\frac{1}{7}$ of $\frac{5}{9}$ of 42 times $\frac{1}{2}$ as much as A. They gained a sum equal to $\frac{2}{3}$ of their joint stock. How much did they gain, and what was the share of each?

23. If Mr. Walton's black-board were 2 ft. wider than it now is, it would contain 26 more square feet, but if it were 2 ft. longer it would contain 11 more square feet. How many square feet does it contain?

24. George has money enough to buy $2\frac{1}{2}$ quarts of chestnuts, Rufus has twice as much as George, and Edward has $\frac{1}{3}$ as much as Rufus. They all have 57 cents. How much are the chestnuts worth per quart, and how many cents has each of the boys?

25. The interest of Mr. Butler's money for 5 yr. 6 mo. 20 da., at 6 per cent, will equal $8000. How much money has he?

26. If a pound of rice is worth $\frac{2}{3}$ as much as a pound of sugar, and 6 lb. of rice and 10 lb. of sugar are worth $1.26, how much are 5 lb. of rice and 7 lb. of sugar worth?

27. Why is it that if we multiply any number whatever by 3, add 7 to the product, add the first number taken to *this, add* 9 to this, divide this by 4, add 3 to this, and then

subtract from this the first number taken, the result will always be 7?

28. By selling cloth at $1.25 per yard, I lose $16\frac{2}{3}$ per cent. For how much per yard must I sell it to gain 20 per cent?

29. There are $\frac{2}{3}$ as many acres in my orchard as there are in my pasture, and $\frac{1}{4}$ as many in my garden as in my orchard. If there are 17 acres in all, how many are there in each lot?

30. I bought a lot of goods for $600, and after keeping them 1 month 17 days, I sold them for $650. Now, allowing that I had to pay interest on the money invested, at the rate of 6 per cent, what was my net gain?

31. A man bought a cask of wine, but $\frac{2}{3}$ of it leaked out. He put in $\frac{1}{2}$ as much water as there was wine remaining, and sold the mixture for $\frac{4}{5}$ as much per gallon as he gave for it. What part of the cost did he lose?

32. After paying $3 more than $\frac{1}{2}$ of my money to one man, and $6 more than $\frac{1}{2}$ of what I had left to another, I had $7 left. How much did I have at first?

33. I sold 10 bushels of corn for Mr. Austin, and 8 bushels for Mr. Brown, receiving $11 for the lot. Now, allowing that Mr. Austin's corn is worth 20 per cent more per bushel than Mr. Brown's, and that I am to receive $1 for my services, how much money ought I to pay to each?

34. A sheep is worth $2 more than a calf, and a cow is worth the sum obtained by adding the value of a sheep to $2\frac{1}{2}$ times the value of a calf. Moreover, a sheep and a calf and a cow together are worth $6\frac{1}{2}$ times as much as a calf. What is the value of each?

35. Adin is 2 years younger than Seth, and Asa is $\frac{3}{4}$ as old as Adin. Now if the difference between Asa's age and

Seth's age be added to Seth's age, the sum will be $1\frac{1}{2}$ times Adin's age. What is the age of each boy?

36. After selling 6 acres more than $\frac{2}{5}$ of my land, I have $\frac{3}{4}$ of $4\frac{1}{2}$ times 12 acres left. How many acres did I sell?

37. John has \$20. John and William together have twice as much as George, and John and George together have 3 times as much as William. How many dollars has each?

PARTIAL SOLUTION. If John's \$20 plus William's money = twice George's money, then once George's money must = $\frac{1}{2}$ of \$20 plus $\frac{1}{2}$ of William's money, = \$10 plus $\frac{1}{2}$ of William's money. But George's money plus John's \$20 must equal \$30 plus $\frac{1}{2}$ of William's money, which by the conditions of the problem is equal to 3 times William's money. Hence \$30 = $2\frac{1}{2}$ or $\frac{5}{2}$ times William's money. If \$30 = $\frac{5}{2}$ of William's money, $\frac{1}{2}$ of William's money must equal, &c.

38. I have 66 marbles. If I give them to John he will have 3 times as many as George, but if I give them to George he will have 4 times as many as John. How many marbles has each boy?

39. I bought a lot of corn, and $1\frac{1}{5}$ times as much oats, for \$10.80. The oats cost $\frac{2}{3}$ as much per bushel as the corn, and there were 22 bushels of both. What was the price of each per bushel?

40. I spent $\frac{1}{9}$ of my money, and earned \$3 more than $\frac{1}{4}$ as much as I had left, when I had \$18 more than $\frac{5}{8}$ of what I had at first. How much did I have at first?

41. Moses has spent $\frac{3}{8}$ of his money, but if he had spent \$1 less than $\frac{1}{2}$ as much as he did, he would have had left \$25 more than he now has. How many dollars does he now have?

42. I offered a dollar-bill in payment for a book, and

found that $\frac{2}{3}$ of the change which I received back was equal to just $\frac{3}{8}$ of the cost of the book. What was the cost of the book?

43. A man has 8 cents more than enough to give a party of beggars 3 cents each, and 6 cents less than enough to give them 5 cents each. How much money has he?

PARTIAL SOLUTION. The difference between the money required to give them 3 cents each and 5 cents each is 14 cents; and as it makes a difference of 2 cents to each beggar, there must have been 7 beggars.

44. Maria's father gave her some peaches, which she wished to distribute among her playmates. She found that if she gave them 5 each she should have 7 left, but that if she had 9 peaches more she would have enough to give them 6 each. How many peaches did her father give her?

45. A farmer sold a lot of hay at $15 per ton, and determined to buy a certain number of sheep with the money received for the hay. He found that if he should pay $10 each for the sheep, he would not have money enough by $12 to buy as many as he wanted. He therefore bought the sheep at $7 each, when he had $30 left. How many tons of hay did he sell?

46. Two persons have equal sums of money, but if one earns $5\frac{1}{4}$, and the other spends $6\frac{2}{3}$, the first will have twice as much as the second. How many dollars has each?

47. A man who had received a large legacy, invested $\frac{1}{3}$ of it in real estate, $\frac{1}{5}$ of it in bank stock, $\frac{3}{10}$ of it in railroad stock, and then put the rest at interest. Moreover, what he put at interest was $6000 less than $\frac{1}{2}$ of what he invested in other ways. What was the value of the legacy?

48. A, B, and C formed a partnership. A put in $3 as often as B put in $2, and C $1; A's money was invested

only $\frac{1}{2}$ as long as B's, and B's was invested $\frac{2}{3}$ as long as C's. They gained $92. How many dollars ought each man to receive?

49. If Erastus can do a piece of work in 9 days, and Hiram can do it in 8 days, what part of it would both do in 1 day, and how many days will it take both to do it?

50. Walter and Reuben together can do a piece of work in 18 days, and Walter can do $1\frac{1}{2}$ times as much work per day as Reuben. How many days would it take each of them working alone to do it?

51. David is $\frac{2}{3}$ as old as John and $\frac{3}{4}$ as old as Henry, and $\frac{1}{2}$ of the sum of their ages added to Henry's age equals 39 years. What is the age of each boy?

52. A man bought a watch, a chain, and a key. The chain cost 3 times as much as the key, and the watch cost $5 more than $2\frac{3}{4}$ times as much as the chain and key together. If the cost of the whole was $113 more than $\frac{1}{2}$ the cost of the chain, what was the cost of each?

53. If 2 oranges cost 1 cent less than 3 lemons, and 8 oranges and 6 lemons cost 32 cents, what will 6 oranges and 8 lemons cost?

54. A man has an eighteen-gallon cask of wine and two empty casks, one of which will hold 7 gallons and the other 11 gallons. How can he by using only these three casks measure out 9 gallons of wine?

THE END.